

Alexander Givental. AS A MATTER OF THOUGHT

I intend these notes for the 12th of June, 2010, the 73rd birthday of Vladimir Igorevich Arnold, and day ten since June 3, when he passed away. I am writing with pain.

In recent years, Arnold would often emphasize that he is more a naturalist than a mathematician. Perhaps, being his true pupil, I cannot help wondering: Why pain? Why not simply regrets or pity? What exactly makes these tragical circumstances painful? Obviously, it is a huge loss, and it is clear of *whom*: of a person close to many of us; but *what* exactly are we missing that makes the pain so persistent?

The last of my memories of Vladimir Arnold is filled with details about that sunny Paris Sunday, April 25, when he met a group of us – including my wife Laura, our son Emile, two Svetas (my sister, and our common friend – the author of his photographic portraits), Julia and Arkady Vaintrob with their daughter Liza – and, after a deliberate outburst of theatrical rudeness (“On the first Spring day like this, 20+ Celsius, I am supposed to start my swimming season rather than be here with you”), gave us - plus Olya Kravchenko, who arrived 'suddenly' (like a train that's late) from Lyon to join the procession with her backpack and two daughters - the long-awaited guided tour through *Jardin de Luxemburg*. We were walking through the park, which most of us have visited dozens of times, and discovering, with his help, dozens of unfamiliar attractions and new features of familiar ones. There was the miniature Statue of Liberty (the prototype of the two more famous clones), a tiny oak-tree planted in the memory of the victims of September 11, the Paris meridian allegedly aligned with 'the center of Paris' spot in front of Notre Dame, a sculpture by Ossip Zadkine hidden near the playground where Emile used to play several years ago, and a lot more. Arnold was in excellent 'shape': encyclopaedic, energetic, articulate - as usual - and in his cheerful mood too.

If we are ever back in Paris, I am sure we will walk through *Jardin de Luxemburg* trying to recall all of that, and we will miss *him*. It feels unjust, that this man, full of enormous living and intellectual energy and plans, suddenly passes away, and we are sad that he disappears from *our* lives. But what exactly do we miss? Is this simply our selfishness that expects more sunny days like that, more entertaining stories about Paris, and we are in pain and anger that we cannot have them any more? This does not sound right, and hence the question that bothers me: What drives this emotion? And generally, what governs our emotions and why? Why would they exist, if not as a means to form and achieve our goals? If so, then which goals in general and in this case in particular?

Clearly, we first react, as infants, to food and comfort stimuli provided by our parents, and acquire in this way our basic survival skills. Then, through a more complex hierarchy of stick-and-carrot programming, we become self-motivated enough to dutifully dedicate years of exhausting work, for example, with a single mathematical theorem at the output. What is that gratification scheme that makes such a gigantic leap of conscience and perseverance possible?

Here is a plausible scenario. We submit our deeds to the judgment of our environment. It includes people around us: parents, teachers, friends, spouse, children, our siblings, other relatives, and everyone else who cares to respond. This environment reacts in the form of materially relevant carrots and sticks, causing the respective positive and negative emotions, and accompanied with gestures of approval or disapproval. This training occurs repeatedly, until Pavlovian conditional reflexes are born. When they are, certain emotional circuits, so to speak, are created, and we fall into a dependency on the gestures *per se*. That is, we begin to seek approval of the person whose judgment became so relevant to

us, because that is what triggers that circuit to provide positive emotions of joy, pride, happiness, etc. and we dread his disapproval, although the gesture itself may be inconsequential. A fashionable yet inaccurate term *role-models* comes to mind, but I prefer to think of these people as our supporting *pillars*. In the hierarchy of pillars, our parents are among the most important ones, and that is probably why, as adults, we continue, even if subconsciously, to seek their appreciation.

Just as Arnold taught us, everything is inter-related, and in reality separating material from the gestural, the training of the gratification circuit from its reflex-like engagement, would be futile. It begins with exams you have to pass to become his student, but you also get simply enchanted by the landscape outlined in the papers you were expected to master, such as his Vancouver's talk at ICM-74 (smuggled out by Varchenko and delivered by Brieskorn). In his seminar, when he asks a control question and then looks for the answer, browsing the room with his eyes, you hide yours or look straight, expressing thereby the degree of your confidence – a habit that comes in handy when control questions are asked in *Jardin de Luxemburg*. You receive from him carefully written pages with concrete and manageable problems for you to solve – and having succeeded, you wonder: How could he ever come up with such a conjecture without knowing the proof beforehand? (Later, you figure out there are ways.) You give talks on his seminar trying to make first and foremost *him* understand your theorem. And he does, by finding his own proof, or generalizing your result, and then makes you publish alone this actually joint work. (“Sasha, once I co-authored a research paper, and it turned out to be wrong, although everything written by each co-author was correct.”) And there is a dozen of such *yous* at a time, and he is the only one there who really understands how theorems of each of them fit together.

He teaches you how to write your papers by editing them and completely rewriting some pages. Later, he starts inviting your comments to his papers, and you are surprised that he accepts most of your corrections (“The reader is always right”); or he asks you to answer 'as the expert' to his questions, and it is rewarding if your answer is also aesthetically pleasing *to him* (“That's exactly the way I like – thank you!”) You feel proud when he starts quoting your results in his talks, lecture courses, books, and survey papers. Then he begins to commission *you* for writing such papers.

You ask if he'd come to your undergraduate thesis defense, and he comes (!) and gives a short and impressive speech. And he gives you a mock-exam (“What! You don't know *this*, and you hope to enter the graduate school of the Moscow State University?!”), and he sits on your exam in *The History of The Party*, making sure you get a grade at least as high as deserve. When needed, he engages his connections in Moscow to help you find jobs, and when the time comes, he sends abroad recommendation letters on your behalf, and feels responsible for your performance. And there are some three dozen or more of such *yous* overall, Russian-speaking or not.

Afterwards, you continue meeting with him once or twice a year here and there, often in the enthusiastic crowd of such as yourself and listen to his anecdotes which become the basis of your common “children's folklore” of this crowd of “siblings.” It is engaging to hear and learn mathematics from him, but it gradually becomes harder and harder to explain *to him* what *you* are doing. Suddenly, in an accident, he receives gravely dangerous injuries, from which he probably never fully recovered. He becomes more gruesome, irritable, at times hard to deal with, sometimes unreasonable, or at least appears so (as to a teen, his “old folk” always does). He grows older, but becomes even more prolific than ever, so this time *you* lose any chance to follow (not excluding “children's folklore,” which becomes darker). However, all these changes in the form, frequency, mode or mood of interaction with Arnold don't change anything, really. For, the gratification circuit, the key reference point, the supporting pillar, has already been there for ages. It is the sudden disappearance of this pillar that causes the pain.

This may be a figure of speech true for everyone, that without *him* the World won't be the same, but I am talking here about something else: the truly emptied space in the inner worlds of those who developed this form of dependency, the habit to trigger this circuit, to rely on this pillar. What can they do about this?

I think we all know the answer.

In recent (or even not so recent) years, Arnold developed an unpleasant habit of refusing an appointment to most of those who wanted to tell him about their theorems or ask how to solve their problems. Apparently it was a necessary defensive reaction: Imagine how numerous were those who would be happy to earn his appreciation of their work! According to an anecdotal story told by Arnold, once he was approached in the middle of a river he was swimming across.

In fact, close students of Vladimir Igorevich learned not to overburden him with their mathematics. In order to engage the gratification circuit, they did not really need to hear him express his reaction to their work, for they *knew* in advance what such reaction would, or at least should, be. In the process of stick-and-carrot training, we learn how our pillars react. Parents *instill values* in their children, but I would rather say that we acquire our *inner voices*. Our pillars/judges are not bosses to whom we submit written yearly reports, but people who respond when we address them in a *gedanken* fashion. The imagination is a marvelous feature of our mind that allows us to engage the gratification circuits by merely talking to our idols and earning their approval in our thoughts.

Perhaps, this explains religion: God is an immortal pillar of universal expertise. As a person, He is never there, but with proper training, one can learn to talk to Him in one's inner voice so that He would always respond. To a religious person, God might be not so much a watchful eye restraining that person from actions evil to the environment, but more of a universal guarantee that someone is always there to get the gratification circuits engaged. Bach was composing in the name of God.

This scheme of circuits and pillars occurred to me all at once, when Arnold was past his surgery, still alive but already sentenced. For a moment, I was entertaining the thought, that the paper I was working on was good enough to be dedicated to him, and then suddenly realized that it made no sense to me at all to dedicate papers *to his memory*. For, the only reason for dedicating a paper to him used to be, for me, the *mere possibility* that he would read – not the paper – the dedication, and reasonably conclude that the theorem was 'good enough.'

What shall we do about this sudden change of options? I think I already gave the answer. We will have to grow up to continue - in our thoughts, just as we did before, but without that *mere possibility* - to lean on this pillar.

This is much easier for us, his students, who got used to consulting with him most of the time only in our mind. It is much harder for those who are used to relying on him – or letting him rely on them – day after day. Yet maybe we can offer something to console them.

The need to earn the approval of our inner judges is not the only driving force of the gratification scheme: There must be another side to this coin. Our thirst for fame and glory – what is it if not the desire to become ourselves pillars and idols? That is probably why we care so much for our children: in the hope, even if ever elusive, to raise at least someone idolizing us at the end. The presence of the crowd of those who consult us in their thoughts even after we are gone is probably the true meaning of the afterlife, and the size of this crowd is a true measure of our legacy.

In old Russia, children and parents would address each other by formal You. This tradition has been preserved in the relationship between Vladimir Igorevich and his students. It is, of course, a common metaphor that compares teachers and students with parents and children. There are many such scientific children of Vladimir Arnold. His *scientific* heritage, that of his textbooks, theorems, problems, conjectures, notions introduced by him or named after him, will be multiplied by their work and by the work of their students and, albeit subject to paradigm shifts, will persist in history. Yet this is not the whole story, not that legacy I am talking about.

There are many of us who are now in pain, and for whom therefore that teacher-parent (or maybe classmate-brother) metaphor was not quite a metaphor, really. This means that there are many of us who feel the loss of the supporting pillar, those who have somehow not only (or not necessarily) digested some of Arnold's theorems, but also inherited some of his opinions and principles, his mathematical taste, teaching habits, writing style, physical culture, his interest in history or poetry, his knack for storytelling, his attention to nature, or his daring recklessness. We will somehow overcome the pain, and will continue, inadvertently, to consult with him in our thoughts and judge our theorems and actions by the criteria we have en-trained into our gratification circuits. Alongside our conscious memories, we will pass on, even if unintentionally, some of these criteria, to our scientific children as well as biological ones. Our current pain assures the lasting afterlife of Vladimir Arnold, of his unique and enormous personality.

Before I can finish these eclectic notes and let them go, I need to decide if what is said here merits the occasion. How would Arnold himself have reacted? Would he say: “It is trivial, everyone knows this,” or issue a forgiving “Thank you, Sasha” with a slightly condescending smile? Would he be pleased? Would he simply say: “It is interesting,” and - the sign of true interest! - start looking for counter-examples?

I would not be writing these notes if I did not think that he *might* have appreciated them. Hope and ambiguity of real life are more merciful than the absoluteness of mathematical precision, and the choice is truly mine. For, with the cruel mathematical certainty, I know one thing: Arnold won't read this.