

A SELFISH REQUEST

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As the author's math-ignorant daughter and a full-fledged graduate of the California education system is applying to PhD programs in comparative literature, her revengeful parent is compiling these notes in the aforementioned genre.

The subjects for the comparison are *Singapore* and *California*, 5th grade textbooks:

Primary Mathematics 5A, US Edition, Federal Publications, Ministry of Education, Singapore ¹, and

Mathematics, book 5, California Edition, Houghton Mifflin Company. ²

Figures 1 and 2 reproduce page 37 from *Singapore* and page 332 from *California* respectively, representing the same topic chosen almost randomly.

In Figure 1, a boy from *Singapore* tells us a short story requiring the addition of $1/3$ and $1/2$. The square prompts invite us to feed in the answer $5/6$. This was not hard, but the question remains: how did the boy guess to replace the fractions with respectively $2/6$ and $3/6$? Is there a secret, a trick? Is he a genius? No, the boy explains, it was a stroke of luck: as the picture shows, the cake was *precut* into 6 equal parts, of which Ann took 2 and her brother 3. Now the general idea is revealed: the problem of adding $1/3$ and $1/2$ looked hard because the denominators were different, but using equivalent fractions with the same denominator makes the problem easy.

In the next two pages, a girl and the boy will help us examine equivalent fractions in 3 more addition and 3 subtraction examples with common denominators climbing up to 30. These will be followed by 3 addition and 3 subtraction exercises to be solved on our own, and respectively — by two

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pointers to homework sets from Workbook 5A. A page of *Practice* with 12 more exercises and several word problems will conclude unit 2. *Addition and Subtraction of Unlike Fractions.*

2 Addition and Subtraction of Unlike Fractions

Ann ate $\frac{1}{3}$ of a cake.

Her brother ate $\frac{1}{2}$ of the same cake.

What fraction of the cake did they eat altogether?

$$\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6}$$

$$= \blacksquare$$

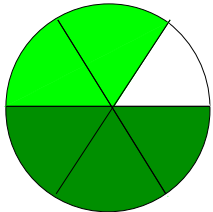
They ate \blacksquare of the cake altogether.

$\frac{1}{3}$ and $\frac{1}{2}$ do not have the same denominator.


They are called *unlike fractions*.

$\frac{2}{6}$ and $\frac{3}{6}$ have the same denominator.

They are called *like fractions*.



The cake is divided into 6 equal parts. Ann ate 2 parts, and her brother ate 3 parts.



We can change unlike fractions to like fractions using equivalent fractions:

$$\frac{1}{3}, \frac{2}{6}, \dots$$


$$\frac{1}{2}, \frac{3}{6}, \dots$$


FIGURE 1

We note the precise and economical character of the text: not a sign is wasted.

On the contrary, the multicolor Figure 2 from *California* asks for an editor's red pen.

LESSON

14

Add Fractions With Unlike Denominators

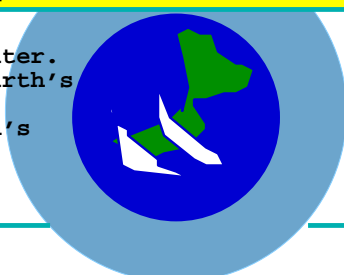
You will learn how to add fractions which have different denominators.

Review Vocabulary
equivalent fractions

Learn About It

Most of Earth's surface is covered by water. The Pacific Ocean covers about $\frac{1}{3}$ of Earth's surface, and the Atlantic Ocean covers about $\frac{1}{5}$. What fractional part of Earth's surface is covered by these two oceans?

Add. $\frac{1}{3} + \frac{1}{5} = n$



Find $\frac{1}{3} + \frac{1}{5}$.

Use the product of the denominators to write equivalent fractions with a common denominator.

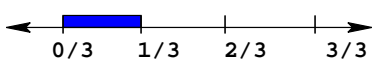
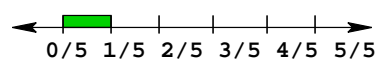
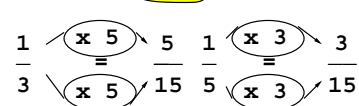
<p>Step 1. Use number lines to model the fractions. Notice that the fractions are different unit lengths.</p> <div style="margin-top: 10px;">  </div> <div style="margin-top: 10px;">  </div> <p>To add fractions $\frac{1}{3}$ and $\frac{1}{5}$, you need to first find equivalent fractions with like denominators.</p>	<p>Step 2. Use the product of the denominators to write equivalent fractions with like denominators.</p> <p style="margin-top: 10px;">$3 \times 5 = 15$ ← common denominator</p> <div style="border: 1px solid black; border-radius: 50%; padding: 5px; margin: 10px auto; width: 80%; text-align: center;"> <p style="margin: 0;">Think: Multiply by the denominator of the other fraction.</p> </div> <div style="margin-top: 10px;">  </div>	<p>Step 3. Rewrite the problem using fractions. Then add.</p> $\frac{1}{3} + \frac{1}{5} = \frac{5}{15} + \frac{3}{15}$ $= \frac{8}{15}$
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FIGURE 2

The opening promise “You will learn how to add fractions which have different denominators” only reiterates the title (or does it? — we will come to this later) and can be safely omitted. The satellite picture of the Earth is of no use and better be dropped too. The scientifically true fact that “Most of

Earth’s surface is covered by water” does not really follow from $8/15 > 1/2$ and, being presently irrelevant, should be removed as well.

“Find $1/3+1/5$ ” is a perfect mathematical formalization of the problem and stays. A cyborg’s thought process “Add. $1/3 + 1/5 = n$ ” reads “a third and a fifth add up to n ” and goes, since it refers to an n which has not been introduced (nor is going to show up later).

My limited English does not allow me to “Notice that the fractions are different unit lengths”. Fortunately the entire Step 1 is redundant: drawing the fractions on the number line does not facilitate the addition.

“Use the product of the denominators to write *equivalent fractions* with a common denominator” explains the plan perfectly and leaves no reason to repeat it in Step 2. Likewise, the instruction “Rewrite the problem using fractions. Then add” in Step 3 adds nothing new after “Find $1/3+1/5$ ”. Removing it also helps one to realize that there is no need to chop the solution into “steps”.

The result of our editing, shown in Figure 3, matches Figure 1 in clarity and simplicity. Yet something still displeases the ear, doesn’t it? Who the heck are these *unlike denominators*?

In *Singapore* (and most of the world), *unlike fractions* have different denominators. Respectively, *like fractions* have equal denominators and are in this sense similar, or “friendly” (as some teachers put it), “speaking the same language” of sixths or fifteenths. *Like fractions* are not necessarily equal, so the word comes handy. Embarrassingly, in *California*, the scholarly term *unlike denominators* stands simply for different ones, so that *like* means nothing but *the same*.³

One can deepen the comparison by noting the variance in the methods of addition of fractions in *Singapore* and *California*: the mental scan of equivalent fractions until they become “friendly” often yields smaller denominators than the product routine. In fact the next Lesson in *California* introduces Least Common Denominators and uses prime factorization, while the *Singapore* math program postpones studying prime factorization until grade 7. One may debate if this makes *California* ultimately more advanced, or argue that in practice the method in *Singapore* is just as efficient, or probe educational advantages of either approach.

One may further discuss how wise it is to fake scientific applications and pretend doing algebra, or try to guess the consequences of replacing ideas with algorithmic “steps”. One may wonder what role is left to thinking when the

³A Russian saying comes to mind “Слышу звон, да не знаю, где он,” which can be translated (thanks to Alisa Givental) as “The tongue speaks, but the head doesn’t know.”

command *think* is used as an euphemism for *do*, or why *Singapore* students don't get a separate subtraction unit while *California* students need it.

LESSON

14

Add Fractions With Unlike Denominators

The Pacific Ocean covers about $\frac{1}{3}$ of Earth's surface, and the Atlantic Ocean covers about $\frac{1}{5}$. What fractional part of Earth's surface is covered by these two oceans?

Find $\frac{1}{3} + \frac{1}{5}$.

Use the product of the denominators to write **equivalent fractions** with a common denominator.

$3 \times 5 = 15 \leftarrow$ common denominator

Multiply by the denominator of the other fraction.

$$\frac{1}{3} \begin{array}{l} \xrightarrow{\times 5} \frac{5}{15} \\ \xrightarrow{\times 5} \frac{5}{15} \end{array} \quad \frac{1}{5} \begin{array}{l} \xrightarrow{\times 3} \frac{3}{15} \\ \xrightarrow{\times 3} \frac{3}{15} \end{array}$$

$$\frac{1}{3} + \frac{1}{5} = \frac{5}{15} + \frac{3}{15}$$

$$= \frac{8}{15}$$

FIGURE 3

All these subtleties are entirely beside the point, which is: *California* is poorly written, period. The book is on the list of instructional materials adopted by the California Department of Education in 2001 and features links to California Math Standards pagewise, yet it is grossly redundant, full of

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irrelevant details, misleading explanations, confusing comments, distracting pictures, embarrassing mistakes.

Dear fellow mathematicians,

On those rare occasions when you are given the role of Content Reviewer of a school textbook, please — may it even be the last sum of cash you receive for such services — let common sense be your guide and the red pen your weapon!

I need your courage and sacrifice: my son has just entered the California education system.