Math 53M Review for Midterm 1 Written by Matt

Problem 1 *Decide if the following are true or false.*

(a) *If* $\vec{a} \neq 0$ *then* $\vec{a} \cdot \vec{b} = 0$ *implies* $\vec{b} = 0$.

(b) *The cross product of any two nonzero vectors is nonzero.*

(c) $\left(\vec{a} \times \vec{b}\right) \cdot \vec{c} = (\vec{c} \times \vec{a}) \cdot \vec{b} = \left(\vec{b} \times \vec{c}\right) \cdot \vec{a}$.

(d) $c\vec{a} \times \vec{b} = \vec{a} \times c\vec{b}$.

(e) $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

(f) $|\vec{a}| > |\vec{b}|$ *if and only if* $\left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right) > 0$.

Problem 2 *Let* O *be the origin and consider the points* $P\,(1,1,0)$, $Q\,(0,1,1)$, $R\,(1,2,1)$, $S\,(1,0,1)$, $T\,(2,1,1)$, $U\,(1,1,2)$, *and* $V\,(2,2,2)$. *Find (noting that the vertices are not listed in "geometric" order)*

(a) *the area of the parallelogram* $OPQR$.

(b) *the volume of the parallelepiped* $OPQRSTUV$.

(c) *the volume of the tetrahedron* $OPQS$.

Problem 3 *What is the angle between*

(a) *the plane* $y = 0$ *and the line* $x - 7 = y - 1 = \frac{z}{\sqrt{2}}$?

(b) *the two planes* $x+y+z = 10$ *and* $\left(1 + \frac{1}{\sqrt{6}}\right) x + \left(1 + \frac{1}{\sqrt{6}}\right) y + \left(1 - \frac{2}{\sqrt{6}}\right) z = 0$?

Problem 4 *For points* $P\,(1,2,0)$, $Q\,(0,1,-1)$, *and* $R\,(1,1,1)$ *find*

(a) *symmetric and parametric equations of the line* PQ.

(b) *the distance from this line to the point* R.

(c) *the area of* $\triangle PQR$.

Problem 5 *Give symmetric equations for the line of intersection of the plane* $x + 3y + 7z = 9$ *with*

(a) *the* yz*-plane.*

(b) *the plane* $x + y + z = 1$.

Problem 6 *Give the equation of the plane containing the line with parametric equations*

$$x = 3 - t \qquad y = 1 + 2t \qquad z = 5$$

and parallel to the line with symmetric equations

$$x = \frac{y-1}{4} = 2z.$$

Problem 7 *Make a detailed graph of the curve whose equation in polar coordinates is* $r = 1 + \cos\left(\theta - \frac{\pi}{3}\right)$.

Problem 8 *Identify the surface*

(a) *whose equation in spherical coordinates is* $\rho = \cos\phi$.

(b) *whose equation in spherical coordinates is* $\sin\phi\,(\cos\theta + 2\sin\theta) - \cos\phi = 0$.

(c) *whose equation in cylindrical coordinates is* $z^2 - r^2 = 4$.

(d) *whose equation in rectangular coordinates is* $\operatorname{proj}_{\vec{a}}(\langle x, y, z\rangle) = 5\vec{a}$.

(e) *whose equation in rectangular coordinates is* $x^2 + y^2 = 3z^2$.

(f) *whose equation in rectangular coordinates is* $z = x^2 - xy$.

(Hint: First do the substitution $s = x$ *and* $t = x - y$*. Then do the substitution* $s = u + v$ *and* $t = u - v$*.)*

Problem 9 *Consider the cone* $2x^2 + y^2 = z^2$.

(a) *Describe the intersection of the cone with the xy-plane.*

(b) *Give the equation of a plane containing the point* $\left(-1, 0, \sqrt{2}\right)$ *and parallel to the plane* $z = \sqrt{2}x$*, a "cylinder" over one of the lines in the previous part.*

(c) *The plane and the cone intersect in a curve. We can tell what kind of curve this is by considering its projection to the xy-plane. What is the equation for this projection? What type of curve is it?*

Problem 10 (Rotated axes) *Consider the cone* $z^2 = 2u^2 + 2v^2$.

(a) *Describe the intersection of the cone with the plane* $u = v$.

(b) *Give the equation of a plane containing the point* $(1, -1, 2)$ *and parallel to the plane* $z = u + v$.

(c) *The plane and the cone intersect in a curve. What is the equation of the projection of the curve to the uv-plane? What type of curve is it?*