

Putnam-1988. Hints

A1: Use symmetries $x \mapsto \pm x$, $y \mapsto \pm y$.

A2: $2xg + g' = 2xg'$

A3: $a/\sin a = 1 + a^2/6 + o(a^2)$

A4: (a) Start with 2 colors. (b) Color infinite grid paper mod 3.

A5: Solve recursion equation: $x_0 = a, x_1 = b, x_{n+1} := 6x_{n-1} - x_n$.

A6: If $\begin{bmatrix} \lambda_1 & \dots & 0 \\ & \dots & \\ 0 & \dots & \lambda_n \end{bmatrix} \begin{bmatrix} 1 \\ \dots \\ 1 \end{bmatrix} = \mu \begin{bmatrix} 1 \\ \dots \\ 1 \end{bmatrix}$, then $\lambda_1 = \dots = \lambda_n = \mu$.

B1: $(u+1)(v+1) = uv + u \cdot 1 + v \cdot 1 + 1$

B2: Show that if $y > 0$ and $y(y-1) > x^2$, then $(y+t)(y+t-1) > (x+t)^2$ for all $t > 0$.

B3: Given $\alpha > 1$, for each n we have: $\min_{0 \leq d \leq n} |n - d\alpha| \leq \alpha/2$.

B4: Consider set $\{n \in \mathbb{N} \mid a_n^{1/n+1} < 1/2\}$.

B5: Solve this problem over $\mathbb{Z}/2\mathbb{Z}$.

B6: Make linear change $n = km + l$ of discrete time in the sequence t_n .