Putnam-1986. Hints

A1. It is a standard calculus problem where, however, the maximum lies on the boundary of the domain.

A2. Do the long division for $x^{20}/(x+3)$.

A3. According to Evan O'Dorney, if $\tan\alpha = n+1$ and $\tan\beta = n$, then $\tan(\alpha-\beta) = 1/(1+n+n^2)$.

A4. Note that every 2×2-submatrix of A must satisfy the same conditions.

A5. $h_i(x) := f_i(x) - \sum_j c_{ij}x_j/2$ pass Clairaut's test $\partial h_i/\partial x_j = \partial h_j/\partial x_i$ for all i,j.

A6. Differentiating in x at $x=1$ one obtains a system of linear equations, which determines a's in terms of b's, and with the aid of magic finds the answer $f(1) = b_1\cdots b_n/n!$

B1. It seems straightforward.

B2. I think there is a typo: the simultaneous equations were meant to be

$$x(x-1)+2yz = y(y-1)+2zx = z(z-1)+2xy.$$

B3. For $k>0$, if $\delta := h - fg \equiv 0 \bmod p^k$, then $(f+s\delta)(g+r\delta) \equiv h \bmod p^{2k}$.

B4. Reformulate the problem in terms of lattice points with coordinates $(m, \sqrt{2}n)$ enclosed between concentric circles of large radii M and $M+1$.

B5. $f = x^2+y^2+(z+xy/2)^2 - x^2y^2/4$

B6. Find $\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1}$. `Warning:` Typo in the formulation; what is required to prove is $A^TD - C^TB = I$.