George M. Bergman Spring 2005 Math 114

Corrections and clarifications to Ian Stewart's *Galois Theory*, Third Edition

Many of these mistakes were discovered by Ken Ribet's Math 114 class a year ago; a few more were sent to me by Adam van Tuyl of Lakehead University, Ontario. I added more that my class and I found during the Semester; and subsequently, four more, pointed out by Kumar Joag-dev in Fall 2010.

P.xx: In the line before the displayed equation, ''solution'' should be ''solution to''. More importantly, in that equation, the ''3'' before the second radical should be dropped.

P.3, third line from bottom: ''Theorem 5'' should be ''Theorem 24.5''.

P.7, next-to-last display: Cross out the ½ after the = sign.

P.8: The label ''(1.2)'' should be on the line with the first displayed equation, $y^3 + py + q = 0$, not lower down. And in the second displayed equation, ''p = ...'', the numerator should simply be $3b - a^2$. (In effect, the $-2a^3$ shown should be $-2a^2$, which simplifies with the preceding term.)

P.9, line after first display: ''$u + v = q$'' should be ''$u + v = -q$''.

P.10, second display: In both summands, the -18 should be 18.

P.11: In first display, $3a$ should be $3a^2$. In second display, $a^2/4$ should be $a^3/8$. In the middle of the page, line immediately before ''We choose'', $+2uy+$ should be $+2uy^2+$. In line immediately after ''We choose'', $\frac{q}{2}\sqrt{2u}$ should be $q/(2\sqrt{2u})$, and the same correction applies in each of the last two displays in the page. Finally, in display (1.11), $2p$ should be $2p^2$.

P.13, second display (the 4th-power expression): This should be $(\alpha_1 + \alpha_2 - \alpha_3 - \alpha_4)^2$.

P.14, Exercise 1.3: Change ''its prime factorization is of the form'' to ''it can be written in the form''; and after ''where'' insert ''the p_i are primes and''.

P.14, Exercise 1.4: After ''Prove'' add ''without assuming Cardano's Formula''. Also, as in the above correction to p.10, both -18's should be 18.

P.15, Exercise 1.8: After ''when'' add ''p and q are real numbers such that''.

P.15, Exercise 1.11: The first sentence should be ''Let $P(n)$ be the number of length-n strings of 0's and 1's such that all 1's that occur (if any) occur in groups of three or more.''

P.20, line 5: Delete the last two equations (those defining $-\infty \times n$ and $(-\infty)^2$) and put in the equation $(-\infty) + (-\infty) = -\infty$.

P.20, proof of Proposition 2.3: In the display, change $a_{n-1}t^{n-1}$ to $a_n t^n$. In the rest of the proof, change all occurrences of n to m.

P.20, first line of last paragraph: ''Proposition 2.2'' should be Proposition 2.3.

P.21, 8th line from bottom: ''that using'' should be ''then using''.

P.25, sentence after last display, beginning ''Because ...'': The justification given there is incorrect. By doing the preceding calculation carefully one can, however, verify that the function is continuous as a function of two variables. But since this concerns analysis, which is not the subject of this course, don't worry about it; you may simply assume the continuity condition holds.

P.26, display slightly below middle of page, ''$q(t)$ ='': The exponent ^{n} in the first summand should be $^{n-1}$, and the last two coefficients, a_1 and a_0, should be a_2 and a_1 respectively.

P.32, top two lines: You might find what he means clearer if you change ''This'' to ''This result, which we will now state'', and change ''in which f is assumed to be linear'' to ''which is essentially what we get if we put $t - \alpha$ for f in this result''.

P.32, 1st line of 2nd paragraph of proof of Proposition 3.1: Change ''for all polynomials of'' to ''whenever g has''.

P.34, 3rd, 4th lines of proof of Lemma 3.5: There are four occurrences of K; the first three should be k.

P.34, 2nd line before *ALGORITHM 3.6*: 600 BC should be 300 BC.

P.35, two lines above Theorem 3.9: Change ‘‘this chapter’’ to ‘‘this section’’.

P.36, Definition 3.10: After ‘‘*A polynomial*’’ add ‘‘*of positive degree*’’. (This change brings Stewart’s definition closer to the usual one, though they are still not equivalent. It is needed for consistency with what Stewart will do on p.40.)

P.37, line 3: Delete this line (beginning ‘‘5.’’), since this example is not valid after the above correction.

P.37, Theorem 3.12: In the statement, change ‘‘*Any nonzero polynomial*’’ to ‘‘*Any polynomial of positive degree*’’; in the second line of the proof change ‘‘= 0 or 1’’ to ‘‘= 1’’, and in the third line of proof change ‘‘h and K’’ to ‘‘h and k’’.

P.38, Theorem 3.16: In the first line of the theorem, after ‘‘*factorization of polynomials*’’ add ‘‘*of positive degree*’’. In the proof, delete the second and third sentences, ‘‘If all ... Otherwise ...’’. (These concern the possibility that some of the irreducible polynomials may be constant, which is not possible under the corrected definition of irreducibility.) Note that as a result, the argument shows that $r = s$. (Stewart uses this fact in the proof of Corollary 3.18 on p.40.)

P.42, last sentence of §3.4: ‘‘Lemma 13’’ should be Lemma 19.13.

P.42, first line of §3.5: ‘‘is that’’ should be ‘‘in’’.

P.42, last line: $1 \leq a \leq n-1$ should be $1 \leq a \leq n$. (This only makes a difference for $n=1$, but having the correct value for $\varphi(1)$ is essential to results such as that of Exercise 3.13.)

P.43, line after second display: ‘‘Exercise 3.1’’ should be Exercise 3.9.

P.43, beginning of 6th line before Example 3.23: ‘‘is irreducible’’ should be ‘‘is reducible’’.

P.43, 4th line before Example 3.23: $\mathbb{Q}[t]$ should be $\mathbb{Z}[t]$.

P.45, third line after Definition 3.26: ‘‘Figure 1.1’’ should be Figure 3.1.

P.46, Theorem 3.28, first line: Change ‘‘*a polynomial*’’ to ‘‘*a nonzero polynomial*’’.

P.49, first display: The final ‘‘$+5$’’ should be ‘‘-5’’.

P.51, last line: After ‘‘Moreover’’ add ‘‘it is easy to check that L is a subring of $\mathbb{C}$, hence’’.

P.53: In the line above the last display, $\mathbb{C}(X)$ should be $\mathbb{Q}(X)$. In the last display, the denominator should have variables $x_1, \ldots, x_n$, like the numerator. On the next line, ‘‘and y_j’’ should be dropped. And on the final line, ‘‘For a proof see’’ should be ‘‘You will prove this as’’.

P.54, 2nd displayed equation of Example 2: $14i - 2\sqrt{5}$ should be $14i + 2\sqrt{5}$. Hence to complete the argument, the next display should be changed to $14i + 2\sqrt{5} - 2(i + \sqrt{5}) = 12i$.

P.56, Exercise 4.9: In the first line, $\mathbb{C}(X)$ should again be $\mathbb{Q}(X)$. In the last line, $\mathbb{Q}[t]$ should be $\mathbb{Q}[t_1, \ldots, t_n]$.

P.59, top line: By ‘‘constant multiple’’ Stewart means ‘‘scalar multiple’’, i.e., the result of multiplying by a member of K.

P.59, beginning of line 9: Change ‘‘contrary to the definition’’ to ‘‘and of lower degree than p (since the highest-degree terms of p and q were the same, and so cancelled), contradicting our assumption that p had smallest degree among such polynomials’’.

P.60, statement of Lemma 5.8: The two congruences ‘‘$a_1 + a_2 \equiv b_1 + b_2$’’ and ‘‘$a_1 a_2 \equiv b_1 b_2$’’ should be $a_1 + b_1 \equiv a_2 + b_2$ and $a_1 b_1 \equiv a_2 b_2$. Likewise, in the first display in the proof, a_2 should in both places be b_1, and vice versa.

P.62, line 5: ‘‘Chapter 16’’ should be Chapter 17.

P.62, beginning of last line of Theorem 5.11: ‘‘$K(t) \to K$’’ should be ‘‘$K(t) \to K(\alpha)$’’.

P.62, 2nd line of Theorem 5.12: ‘‘*isomorphic to* $K[t]/\langle m\rangle$’’ should be ‘‘*isomorphic to* $K[t]/\langle m\rangle : K$’’.

P.63, Lemma 5.14: Move this to p.68, end of §6.1.

P.70, two lines above middle display: delete the words ‘‘unless $a = b = 0$’’. And in that display, the left-hand side, which is shown as ending ‘‘$]:\mathbb{Q}$’’, should end ‘‘$:\mathbb{Q}]$’’.

P.72, line after Definition 6.10: ‘‘Exercise 6.12’’ should be Exercise 6.11.

P.72, statement of Lemma 6.11: On first line, delete ‘‘*L is algebraic over K and*’’; on second line after ‘‘$\in L$’’ add ‘‘*algebraic over K*’’, and at end of statement add ‘‘*In this case L is an algebraic over K*.’’ In the first two lines of the proof, change ‘‘algebraic extension’’ to ‘‘extension generated by finitely many algebraic elements’’.

P.73, Exercise 6.8: ‘‘if ∂p and $[L:K]$ are coprime’’ should be ‘‘if ∂p does not divide $[L:K]$’’.

P.75, 8th lines from bottom, et seq.: The author uses, here and throughout the chapter, the wording (which I suppose is British school-usage) ‘‘draw a circle, centre O ...’’ where we would say ‘‘draw a circle with center O ...’’.

P.77, Figure 7.2 is poorly drawn: p_1 and p_2 are supposed to be the centers of the two circles.

P.82, top line: ‘‘contructing’’ should be ‘‘constructing’’.

P.83, Figure 7.6: On the horizontal diameter, the label ‘‘B’’ should be moved so that it labels the right-hand endpoint. (So the best place to put it is just outside the circle, like the ‘‘A’’ at the other end.)

P.84, Exercise 7.15: After ‘‘a regular hexagon’’ add ‘‘of side AB = 1’’.

P.84, first line of Exercise 7.16: In the angles named, remove the denominators ‘‘3’’.

P.90, end of third line: ‘‘of maps’’ should be ‘‘of bijective maps’’.

P.90, end of first line of 2nd paragraph: ‘‘(1) and (2)’’ refer to the statements at the top of the preceding page.

P.92, first line of item 3: ‘‘Example 8’’ should be Example 6.8.

P.93, near end of top line: The ‘‘is’’ (after $\subseteq L$) should be deleted.

P.94, line after last display: ‘‘Exercise 7.1’’ should be Exercise 8.1.

P.95, 4 lines above next-to-last display: After ‘‘generated’’ add ‘‘over $\mathbb{C}$’’.

P.95, line after next-to-last display: Change ‘‘the lexicographic ordering’’ to ‘‘an appropriate ordering’’, and at the end of the line change ‘‘Exercise 8.2’’ to ‘‘Exercise 8.4’’.

P.96, last line of Definition 8.8: Change $K_{j+1} = K_j(\alpha_j)$ to $K_j = K_{j-1}(\alpha_j)$.

P.96, end of next-to-last line: Change $\alpha_j^{p_1 \cdots p_l}$ to $\alpha_j^{p_{l+1} \cdots p_k}$ $(l = 0, \ldots, k)$. And on the last line, change the formula shown to ‘‘$\beta_0 \in K_j$ and $\beta_l^{p_l} \in K_j(\beta_{l-1})$’’.

P.97, second line after first display: $N \subseteq \mathbb{A}_n$ should be $N \supseteq \mathbb{A}_n$.

P.98, middle: The end-of-proof sign $\square$ should be one line higher.

P.98, ‘‘PROOF OF THEOREM 10’’ should be ‘‘PROOF OF THEOREM 8.10’’.

P.98, line after last display: $\sigma(\alpha_1) \notin K$ should be $\sigma(\alpha_1) \neq \alpha_1$.

P.99, two lines above 3rd display: Add ‘‘Thus $K_1 = K(\alpha) \subseteq K(\delta)$. But $K(\alpha)$ and $K(\delta)$ are both of degree 2 over K, so in fact they are equal.’’

P.99, line above the ‘‘$\square$’’ in middle of page: ‘‘Lemma 11’’ should be Lemma 8.11.

P.100: Before reading Theorem 8.12, read Definition 15.1 on p.153 (ignoring the phrase ‘‘in $\mathbb{C}$’’). Back in Theorem 8.12, the condition that $F(t)$ ‘‘can be solved by radicals’’ means that each of its roots t_i is contained in a radical extension of K, in the sense of that definition.

P.100, third line above Definition 8.15: ‘‘Exercise 15.14’’ should be Exercise 15.11.

P.100, line after Definition 8.15: Change ‘‘every radical extension has finite height’’ to ‘‘the height of every radical extension is defined’’. (By definition it will, indeed, be finite, but that isn’t the point.)

P.100, Lemma 8.16, end of statement *1.*: ‘‘$\ldots, p$’’ should be ‘‘$\ldots, p-1$’’.

P.101, first full line: $kp + ql$ should be $qp + lk$.

P.101, 3rd line after first display: After ‘‘linear factors’’ add ‘‘over some larger field’’.

P.101, 4th line after first display: After ‘‘less direct.’’ add ‘‘So assume $t^p - a$ were reducible over M’’.

P.101, second display and preceding sentence: In the sentence, ‘‘P and P_j’’ should be ‘‘P_j and P_k’’, and in the display, $cP(t)$ should be $cP_k(t)$.

P.101, 4th line before last display: ∂p should be ∂P. Beginning of line after that display: ‘‘When $\partial p =$’’ should be ‘‘Since $\partial P =$’’

P.101, 3rd line after last display: Change ‘‘not a pth root’’ to ‘‘not a pth power’’.

P.101, last 3 lines, and p.102, first two lines: Don’t take this too seriously, since he has been assuming since the middle of the preceding page that we are given a radical extension R' of K containing L, with no questions asked about where we got it.

P.102, statement of Lemma 8.17: Here R, M and α are assumed as on the preceding page, in the sentence before the last three lines.

P.102, two lines before third display: In the first of these, after ‘‘Therefore,’’ add ‘‘up to multiplication by nonzero elements of M’’. You should expect it to take some nontrivial thinking to verify the assertion of this sentence. In the next line, α^s should be $x_s\alpha^s$.

P.102, last display: under the Π, the $\mathbb{S}$ should be $\mathbb{S}_n$.

P.102, before last line insert:

> ***Lemma 8.18½***
>
> *Suppose $L \subseteq M$ are fields, and p a prime such that L contains a primitive pth root of unity ζ. Suppose moreover that $\alpha \neq 0$ and $x_0, \dots, x_{p-1}$ are elements of M such that L contains the p elements*
>
> $$\begin{array}{l} x_0 + \alpha\, x_1 + \alpha^2\, x_2 + \ldots + \alpha^{p-1} x_{p-1} \\ x_0 + (\zeta\alpha)\, x_1 + (\zeta\alpha)^2\, x_2 + \ldots + (\zeta\alpha)^{p-1} x_{p-1} \\ \qquad \cdot \quad \cdot \quad \cdot \\ x_0 + (\zeta^{p-1}\alpha)\, x_1 + (\zeta^{p-1}\alpha)^2\, x_2 + \ldots + (\zeta^{p-1}\alpha)^{p-1} x_{p-1}. \end{array}$$
>
> *Then each of the elements x_0, αx_1, $\alpha^2 x_2$, ..., $\alpha^{p-1} x_{p-1}$ also lies in L. Hence, if $x_1 = 1$, then α and each of the elements $x_0, \dots, x_{p-1}$ lies in L.*
>
> **PROOF**. For any $m = 0, \dots, p-1$, consider the sum in L of the first element in the above list, ζ^{-m} times the second element, ζ^{-2m} times the third, etc. through $\zeta^{-(p-1)m}$ times the last. Recalling that $1 + \zeta + \ldots + \zeta^{p-1} = 0$, we can verify that each ‘‘column’’ in the resulting sum adds up to zero except the column beginning $\alpha^m x_m$, which adds up to $p\alpha^m x_m$. So that element lies in L, hence so does $\alpha^m x_m$, as claimed.
>
> If $x_1 = 1$, then the $m = 1$ case of this observation gives $\alpha \in L$, hence the fact that each element $\alpha^m x_m$ lies in L implies that each element x_m does. []

P.103, line before last paragraph of proof: Change ‘‘Therefore’’ to:

> Now if we take the equation $m(x) = 0$ and write x as above in terms of powers of α and coefficients in R_1, and consider the result as an equation satisfied by α, this will have the form $f(\alpha) = 0$ where $f(t) \in R_1[t]$. Hence $f(t)$ is divisible by the minimal polynomial of α, which is $t^p - a$. Hence all the roots of that polynomial, namely α, $\zeta\alpha$, ..., $\zeta^{p-1}\alpha$ where ζ is a primitive pth root of unity, are also roots of $f(t)$. Now by the way we just defined $f(t)$, this means all the elements $x_0 + \zeta^j\alpha + x_2\,(\zeta^j\alpha)^2 + \ldots + x_{p-1}(\zeta^j\alpha)^{p-1}$ $(j = 0, \dots, p-1)$ are roots of $m(t)$. Hence these lie in L, and so Lemma 8.18½ shows that

P.103, last paragraph of proof: At the beginning of the paragraph, ‘‘Also, α’’ should be ‘‘Also, α^p’’, and in the third line, ‘‘all of these elements’’ should be ‘‘all of these radical extensions’’.

P.103, two lines above ‘‘[]’’: ‘‘Exercise 8.6’’ should be Exercise 8.11.

P.104, first line of Exercise 8.4: ‘‘Exercise 2.12’’ should be Exercise 8.3.

P.105, Exercise 8.8: $(\alpha_1 - \alpha_2)^2$ should just be $\alpha_1 - \alpha_2$.

P.105, Exercise 8.10: Change the second line to ‘‘show that $\alpha_{\sigma(1)} + \alpha_{\sigma(2)} - \alpha_{\sigma(3)} - \alpha_{\sigma(4)}$ is a Ruffini radical for all $\sigma \in \mathbb{S}_4$.’’

P.105, Exercise 8.12: In parts (c) and (d), after ‘‘the Galois group of’’ add ‘‘any finite algebraic extension’’.

P.107, second paragraph, end of next-to-last and start of last line: ‘‘an extension’’ should be ‘‘a finite extension’’.

Pp.107-109: In the first lines of Definition 9.1, Definition 9.3, Theorem 9.4 and Lemma 9.5 change ‘‘*polynomial*’’ to ‘‘*nonzero polynomial*’’.

P.109, second line of Lemma 9.5: $\supset$ should be $\supseteq$.

P.109, line before last display: ‘‘Theorem 16’’ should be Theorem 5.16.

P.109, last display: The first α_1 should be σ_1. (The display should read $j_1 : K(\sigma_1) \to K'(\alpha_1)$.)

P.111, middle: Drop the ‘‘[]’’ and put it on the next page, line 5 (i.e., end of the first paragraph).

P.111, 5th line from bottom: Cross out the sentence beginning ‘‘Furthermore’’.

P.112, line 2: ‘‘(8.1)’’ should be (9.1).

P.113, end of statement of Lemma 9.13: ‘‘$\Sigma[t]$’’ should be $K[t]$.

P.113, line before Proposition 9.14: ‘‘over $\mathbb{C}$’’ should be ‘‘over subfields of $\mathbb{C}$’’

P.114, last words before exercises: ‘‘all n’’ should be ‘‘all $n > 0$’’.

P.119, sentence before Example 10.2: ‘‘(9.1)’’ should be (10.4).

P.120, statement of Lemma 10.3: In the display, change both subscripts m to i, and add at the end ‘‘$(i = 1, \ldots, m)$’’. In the next line, delete the word ‘‘*complex*’’, and after a_{ij} add ‘‘*in a field* K’’. In the final line, after x_i add ‘‘*are all in* K *and*’’.

P.120, next-to-last word of line after end of proof of Lemma 10.4: Delete ‘‘is’’.

P.121, second line after first display: ‘‘Lemma 1’’ should be Lemma 10.1.

P.121, line before display (10.9): ‘‘Lemma 4’’ should be Lemma 10.4.

P.123, line 6, and again in Exercise 10.1: ‘‘Theorem 5’’ should be Theorem 10.5.

P.126, end of proof of Theorem 11.3: The end-of-proof sign [] should come one line higher.

P.128, top: The end-of-proof sign [] should be at the end of the first paragraph, not the second.

P.128, second line of proof of Theorem 11.9: ‘‘Lemma 8’’ should be Lemma 11.8.

P.129, proof of Corollary 11.11: ‘‘Theorems 11.9 and 11.10’’ should just be ‘‘Theorem 11.10’’. The end-of-proof sign [], presently on the line after that one, should be moved up to that line.

P.129, first line of proof of Theorem 11.12: ‘‘Corollary 11’’ should be Corollary 11.11.

P.130, top paragraph; replace everything after the first sentence (‘‘Let N ...’’) by: ‘‘Then the set of K-monomorphisms $L \to M$ is contained in the set of K-monomorphisms $L \to N$, and by Theorem 11.10 there are precisely $[L:K]$ of those. []’’.

P.130, 3rd-from-last line of proof of Theorem 11.14: Change ‘‘τ, so τ’’ to ‘‘$L \to N$, so τ’’.

P.131, Exercise 11.7(g): After ‘‘fails’’ add ‘‘to be bijective’’.

P.134, two lines above Lemma 12.2: remove the end-of-proof sign []. (Stewart has proved part of the theorem, but won’t finish it till the top of the next page.)

P.133, line before Theorem 12.1: Replace the words ‘‘that is’’ by ‘‘and as we saw in (8.2) (p.93),’’ and at the end of that line, change $H^{*\dagger}$ to $H^{\dagger *}$.

P.135, line 3: There should be an end-of-proof sign [].

P.135, second line of Exercise 12.2: $\subset$ should be $\subseteq$.

P.137, third-from-last display: Note that the British write ‘‘2.4’’ where we write ‘‘$2\cdot 4$’’ and vice versa. This will crop up in a few more places later on.

P.138, Figure 13.1, lower right-hand corner: This should be labeled $-i\xi$ (the negative of the label on the upper left-hand corner).

P.139, upper block-display: In the third and fourth lines, the comma before the final τ should be deleted; i.e., σ^2, τ and σ^3, τ should be $\sigma^2\tau$ and $\sigma^3\tau$ respectively.

P.145, beginning of second display: ‘‘$1 + H_0$’’ should be $1 = H_0$.

P.146, Definition 14.5: After ‘‘*if*’’ add ‘‘*it is nontrivial and*’’.

P.147, first display: Write in the margin *Here and on the next page he composes permutations from left to right*.

P.147, second display: On this line and the preceding, ‘‘$(1b)$’’ should be ‘‘$(2b)$’’ (three occurrences in all).

P.153, last display: ‘‘$(j \geq 2)$’’ should be ‘‘$(j \geq 1)$’’.

P.154, 3rd and 4th lines after Definition 15.2: Change ‘‘it is pointless to expect everything expressible by the same radicals to be inside the splitting field’’ to ‘‘those radicals need not themselves lie in the splitting field’’. And at the end of the last line of that paragraph, change ‘‘Exercise 15.7’’ to ‘‘Exercise 15.6’’.

P.155, change last sentence of proof of Lemma 15.4 to, ‘‘Since α_i is a member of a radical sequence for a subfield of M, so is β_{ij}. By combining such sequences, we get a radical sequence for M.’’

P.155, line after Lemma 15.5: ‘‘We’’ should be ‘‘When $K = \mathbb{Q}$ we’’.

P.156, line between first two displays: Change this to ‘‘we know that ε and η are roots of $t^n - 1$, hence belong to K. Hence’’.

P.156: Delete the line beginning ‘‘Again,’’ after the end of the proof at top of page.

P.156, second line of next-to-last paragraph: ‘‘splits in L because’’ should be ‘‘splits in L. Because’’. (The ‘‘Because’’ introduces the reason for the assertion that follows, ‘‘f has no repeated roots’’, not for the assertion that precedes, ‘‘f splits in L’’.)

P.156, displayed diagram at the bottom of the page: ‘‘Lemma 6’’ and ‘‘Lemma 5’’ should be Lemma 15.6 and Lemma 15.5.

P.157, line after 3rd display: ‘‘Theorem 15.4(3)’’ should be Theorem 14.4(3), and $\Gamma(L:K_0)$ should be $\Gamma(L:K)$.

P.157, next-to-last line of proof of Theorem 15.3: ‘‘Theorem 15.4(2)’’ should be Theorem 14.4(2).

P.159, 4th line before Theorem 15.11: After ‘‘$(1m)(1r)(1m) = (mr)$’’ add ‘‘$(1 < m < r)$’’.

P.161, Exercise 15.1(b): ‘‘)/)’’ should just be ‘‘)’’.

P.165, Example 16.4, part 5: $\mathbb{Q}[t]$ should be $\mathbb{Q}(t)$. (In contrast to part 4, where $\mathbb{Q}[t]$ is correct.)

P.166, next-to-last line: Replace ‘‘*nonempty subset* S *of* R’’ by ‘‘*subset* S *of* R *containing* 0 *and* 1’’.

P.167: Replace the top four lines by:

> *2. A* subfield *of a field* F *is a subring* S *of* F *such that if* $a \in S$ *and* $a \neq 0$, *then* $a^{-1} \in S$.
>
> *3. An* ideal *of a ring* R *is a subset* I *of* R *such that* $0 \in I$, *and if* $a, b \in I$ *then* $a+b \in I$ *and* $a-b \in I$, *and if* $a \in I$ *and* $r \in R$, *then* ar *and* ra *lie in* I.

P.167, first display: To the two equations shown, add a third: $\varphi(1) = 1$; and at the end of the display, change ‘‘for all $r \in R$, $s \in S$’’ to ‘‘for all $r, s \in R$’’. (That the condition $\varphi(1) = 1$ was intended is shown by p.188, parenthetical note in Exercise 17.2.)

P.167, Example 16.6, part 2, first line: $\mathbb{K}[t]$ should be $K[t]$.

P.167, third-from-last line: ‘‘Its kernel is R’’ should be ‘‘Its kernel is I’’.

P.169, last paragraph of §16.3: This seems to refer to a different proof of Proposition 2.3 from the one actually given; namely, one that shows that a polynomial over $\mathbb{C}$ of degree $\leq n$ which has all of $0, \ldots, n$ as roots must be zero. (Key step: Since $f(n) = 0$, we can write $f(t) = (t-n)\, g(t)$, where g has degree $\leq n-1$ and has $0, \ldots, n-1$ as roots, allowing us to complete the proof by induction.)

P.169, next-to-last line: In place of ''*Every prime subfield*'' I recommend ''*For every field* K, *the prime subfield of* K'', as clearer.

P.170, third line after Definition 16.10: Change ''Exercise 16.2'' to ''Exercises 16.5 and 16.6''.

P.171, line 2: ''is an integer'' should be ''as an integer''.

P.172, third line after middle display: ''Exercise 16.3'' should be Exercise 16.7.

P.173, top line: ''Exercise 16.4'' should be Exercise 16.8.

P.178, second paragraph of §17.2: The sentences beginning near the end of the first line (''Simple transcendental extensions...'') and near the middle of the fourth line (''As before...'') say the same thing in different words. I suggest crossing out the former, since the latter gives a bit more detail.

P.178, end of statement of Theorem 17.2: Add ''*In this field,* $I+t$ *is a zero of* m''.

P.179, paragraph before Theorem 17.3: Delete the parenthetical comment on 4th through 6th lines, ''(This is the only place we use the fact that m is monic. ...)'', since he isn't using that fact.

Pp.180-181: Insert ''*nonzero*'' before ''*polynomial*'' in the first line of Definition 17.5, the second line of Definition 17.6, and the first line of Theorem 17.8.

P.181, 2nd line above Theorem 17.8: ''Theorem 5.7 (generalized)'' should be Theorem 17.2.

P.182, proof of Theorem 17.10: Delete the final clause ''except ... a splitting field''.

P.182, last line: ''Lemma 5.9'' should be Lemma 5.14.

P.183, second table: Upper left-hand corner should be marked '$\cdot$' (as in the second table on p.174).

P.184, second display: The term $(p-r)!$ should be in the denominator along with $r!$.

P.184, beginning of second line above Example 17.16: ''7.12'' should be 17.12.

P.184, last line: Before ''have'' insert ''are monic and''.

P.185, end of top line: τ^2 should be $(-\tau)^s$.

P.187, line after second display: Change ''Using the Frobenius map'' to ''Applying the Frobenius map to minimal polynomials''.

P.189, Exercise 17.10: Change ''there exist'' to ''there may exist''.

P.190, Exercise 17.14(a): Change this to ''The minimal polynomial over a field K of any element of an algebraic extension of K is irreducible over K.''

P.190, Exercise 17.14(b): Before ''irreducible'' add ''monic''. (Without this change, this would be a trick question; with it, it tests one's understanding of an important result. My guess is that it was intended in the latter way.)

P.191, Definition 18.2: Change $t_1, \ldots, t_n$ in the first and last lines of this definition to $\alpha_1, \ldots, \alpha_n$, and in the last line change ''*p over K (in n indeterminates)*'' to ''$p(t_1, \ldots, t_n) \in K[t_1, \ldots, t_n]$''.

P.192, replace the proof of Lemma 18.4 by the following:

PROOF The idea will be to show that we can ''replace'' the α's in the given family of independent transcendental elements by the β's, one by one. Precisely, we shall show by induction, for $q = 0, \ldots, r$, that there exist distinct positive integers $i(1), \ldots, i(q) \leq s$ such that the r elements

$$\beta_{i(1)}, \ \ldots, \ \beta_{i(q)}, \ \alpha_{q+1}, \ \ldots, \ \alpha_r \tag{1}$$

are still independent transcendentals over K. For $q = 0$ we understand this to mean that $\alpha_1, \ldots, \alpha_r$ are independent transcendentals, which is given, and thus starts our induction. Note that the $t = r$ step of our induction will give us, in particular, distinct indices $i(1), \ \ldots, \ i(r) \in \{1, \ldots, s\}$, and so show that $r \leq s$. Applying this result with the roles of the α's and β's reversed gives $s \leq r$, and combining these inequalities will yield $s = r$, as desired. Here, now, is the induction:

Assume that for some q with $0 \leq q < r$ we have a list (1) of independent transcendental elements. To construct a similar list for $q+1$, let us first drop α_{q+1} from (1), and consider the field generated over K by the remaining $r-1$ terms,

$$M = K(\beta_{i(1)}, \ldots, \beta_{i(q)}, \alpha_{q+2}, \ldots, \alpha_r).$$

We claim that at least one of $\{\beta_i \mid i = 1, \ldots, s\}$ must be transcendental over M. For if all the β_i were algebraic over M, then $M(\beta_1, \ldots, \beta_s)$ would be algebraic over M; but by assumption L is algebraic over $K(\beta_1, \ldots, \beta_s)$, hence over $M(\beta_1, \ldots, \beta_s)$; so L would be algebraic over M. In particular, our deleted element α_{q+1} would be algebraic over M. Assuming this, let $f(t) \in M[t]$ be a nonzero polynomial having α_{q+1} as a zero. Each coefficient of f lies in M, hence can be written as a rational expression over K in $\beta_{i(1)}, \ldots, \beta_{i(q)}, \alpha_{q+2}, \ldots, \alpha_r$, hence by clearing denominators in these rational expressions, we can assume without loss of generality that each coefficient of f is a polynomial over K in those $r-1$ elements. Hence the equation $f(\alpha_{q+1}) = 0$ can be regarded as a nonzero polynomial relation with coefficients in K satisfied by (1), contradicting our inductive assumption that (1) was independent.

Hence, let $i(q+1)$ be some value such that $\beta_{i(q+1)}$ is transcendental over M. Note that this cannot equal any of $i(1), \ldots, i(q)$, since for each of these indices, the corresponding β is contained in M. Thus, $i(1), \ldots, i(q+1)$ are distinct.

We now claim that

$$\beta_{i(1)}, \ldots, \beta_{i(q+1)}, \alpha_{q+2}, \ldots, \alpha_r \tag{2}$$

are independent transcendentals over K. For suppose there is a nonzero polynomial $p \in K[t_1, \ldots, t_n]$ that becomes zero when these elements are substituted for the indeterminates. If we collect terms of p according to the power of t_{q+1} that appears, we get an expression $p = p_d\, t_{q+1}^d + \ldots + p_1 t_{q+1} + p_0$, where at least one of the coefficients p_i must be a *nonzero* polynomial in $t_1, \ldots, t_q, t_{q+2}, \ldots, t_r$. Since $\beta_{i(1)}, \ldots, \beta_{i(q)}, \alpha_{q+2}, \ldots, \alpha_r$ are independent transcendentals, that coefficient p_i will remain nonzero when these are substituted for the t's; hence on making that substitution, p becomes a nonzero polynomial over M satisfied by $\beta_{i(q+1)}$, contradicting our choice of this as an element transcendental over M. So the elements of (2) are, as claimed, independent transcendentals over K, completing our inductive argument, and the proof of the lemma. □

P.193, end of §18.1: Add

COROLLARY 18.6½

If $K \subseteq E \subseteq F$ are fields, with F algebraic over E, then the transcendence degrees of F over K and of E over K are equal.

This is proved by taking an M between K and E as in the statement of the preceding proposition. It is used implicitly in §8.3.

P.193, next-to-last line: Change ''The expression ... is'' to ''The expressions ... are''.

P.194, beginning of second line: ''evaluated as'' should be ''evaluated at''. I can't figure out the point of the second half of the sentence, ''a minor technical distinction ...'', so ignore it unless you can see the point of it.

P.194, proof of Theorem 18.8: ''Exercise 8.2'' should be Exercise 8.4.

P.195, line before third from last display: Before '', then'' insert ''and $s'_n = s'_{-1} = 0$''.

P.195, first line of last display: The s with no subscript should be s_1.

P.195, last line: The end-of-proof sign □ belongs on the next page, on the line before Corollary 18.10.

P.197, line 4: Move ''[b]y Theorem 11.4'' to after ''is separable and''.

P.197, end of 4th line of §18.4: After ''every quintic'' add ''with coefficients in K''.

P.200, 2nd paragraph: In 1st line, ''Lemma 15.6'' should be Lemma 15.5. In 2nd line, delete ''(generalized)''.

P.200, last paragraph of proof: Note that although in Chapter 13, ''I'' denoted the trivial subgroup, here it denotes a different subgroup, which Stewart specifies.

P.200, middles of last two complete paragraphs: In each case, after ‘‘Theorem 18.17’’ delete the word ‘‘(generalized)’’.

P.202, second display: The term $s_2 t^3$ should be $s_2 t$.

P.205, first line of Exercise 18.6: Delete the words ‘‘of characteristic 0’’, and after ‘‘finite’’ add ‘‘, separable,’’. (This result is needed in prime characteristic for Exercise 18.13.)

P.206, Exercise 18.13: In the second line, after ‘‘finite’’ insert ‘‘normal separable’’, and in the third line delete ‘‘– radical extension –’’. (Even in characteristic 0, such extensions are not what Stewart calls ‘‘radical extensions’’, but ‘‘extensions expressible in radicals’’. Here he means ‘‘extensions expressible in what might be called radicals in a generalized sense’’, which the answer to the exercise is supposed to define.)

P.209, 4th line of 2nd paragraph: ‘‘Example 7.13’’ should be Exercise 7.13.

P.211, 6th line from bottom: ‘‘$4\pi/5$, $4\pi/5$, $2\pi/5$’’ should be ‘‘$2\pi/5$, $2\pi/5$, $\pi/5$’’.

P.212, end of next-to-last line: ‘‘Theorem 4’’ should be Theorem 7.4.

P.214, Figure 19.8: The label $(1,0)$ should be $(-1,0)$.

P.214, line after first display: ‘‘Lemma 2’’ should be Lemma 19.2.

P.214, end of first sentence of Theorem 19.4: Add ‘‘*containing* $(0,0)$ *and* $(1,0)$’’.

P.215, Lemma 19.5: Before ‘‘*finite*’’ add ‘‘*nontrivial*’’.

P.215, end of first line of Proposition 19.7: Add ‘‘*containing* $(0,0)$ *and* $(1,0)$’’.

P.215, 2nd line of proof of Proposition 19.7: Delete the words ‘‘so G is a 2-group’’. (He hasn’t defined the phrase, and in view of his wording of the Corollary 19.6 that comment is not needed.)

P.216, first word of proof of Lemma 19.11: Here ‘‘The’’ means ‘‘Any’’.

P.217, line 3: ‘‘Theorem 19.4’’ should be Theorem 7.4.

P.217, Line before Lemma 19.13: The end-of-proof sign [] should be three lines higher than it is. Also, on the line above where Stewart has that sign, ‘‘The analysis’’ means ‘‘The analysis given below’’.

P.217, next-to-last line of proof of Lemma 19.13: For ‘‘the usual remark about’’ read ‘‘calculating the’’.

P.218, line 3 of proof of Theorem 19.15: ‘‘by Theorem 19.4’’ should be ‘‘by the first sentence of Lemma 19.9’’.

P.225, Exercise 19.11: In parts (h)-(j), after ‘‘constructed’’ and ‘‘constructible’’ add ‘‘from the set $\{(0,0), (1,0)\}$’’.

P.227, third line of proof of Theorem 20.1: ‘‘Theorem 16.1’’ should be Theorem 6.1 (p.67).

P.228, Theorem 20.2: ‘‘$p = p^n$’’ should be $q = p^n$.

P.228, proof of Theorem 20.2: The end-of-proof sign [] should occur one paragraph later than it does, after ‘‘Therefore, $|K| = q$’’.

P.229, second line of proof of Lemma 20.6: Change ‘‘Then G must possess elements g_j whose orders are’’ to ‘‘Then for each j, G must possess an element g_j whose order is’’.

P.231, 3rd line of last paragraph before exercises: After ‘‘$\alpha^3 = 1$’’ add ‘‘so α has order 3 in the multiplicative group of $\mathbb{GF}(4)$’’.

P.232, first line of Exercise 20.5: GF should be $\mathbb{GF}$.

P.235, first two displays: The numerators the final summand of each should be ‘‘1’’, not ‘‘2’’.

P.235, display (21.4): In the big square root, change the $\sqrt{5}$ to $(\pm\sqrt{5})$. In the line after that display, change the words ‘‘with independent choices of the signs $\pm$’’ to ‘‘where the first two $\pm$ signs are independent, and the third agrees with the first’’.

P.237, first display: $(1+2i)$ should be $(-1-2i)$.

P.238, line 6: Starting here, Stewart writes $\mathbb{Z}_n^*$ for the group of units (i.e., invertible elements) of the ring $\mathbb{Z}_n$. (When n is a prime, this consists of all the nonzero elements; for general n it consists of the images of those integers m that are coprime to n.)

P.238, second and third displays: $\rho_2(\zeta)^2$ and $\rho_2(\zeta)^3$ should be $\rho_2^2(\zeta)$ and $\rho_2^3(\zeta)$ respectively in both displays. In the line after the latter display, $\rho_4(\zeta)^4$ should be $\rho_2^4(\zeta)$.

P.238, last line of next-to-last paragraph: By ''that extension'' Stewart means $\mathbb{Q}(i,\zeta):\mathbb{Q}(i)$.

P.239, line 4: ''quotient group'' should be ''subgroup''.

P.239, fourth line below first display of §21.3: ''$\mathbb{Q}(\theta\zeta):\mathbb{Q}$'' should be $\mathbb{Q}(\theta\zeta):\mathbb{Q}(\theta)$.

P.242, line 5 and again on the 2nd line after Theorem 21.3: Before ''$\zeta, \zeta^2, \ldots, \zeta^{p-2}$,'' insert ''1,''.

P.242, first paragraph of proof of the Vandermonde-Gauss Theorem: The end-of-proof sign [] at the end of this paragraph belongs at the bottom of the next page.

P.242, beginning of 5th from last line, ''$\mathbb{Z}_n^*$ is cyclic'': This is true by Corollary 20.9.

P.243, next-to-last display: This formula is to be applied to the result of summing the second display on the page over all values of l, and looking at the coefficient of a given element $\zeta^{\alpha^{j-1}}$ in all summands. Hence the l of this display corresponds to j, not l, in that display; so the computation would be clearer if l were replaced by j in this formula. This in turn would lead to making the same change in Exercise 21.5.

P.243, end of 7th and start of 6th lines from bottom: Where he says ''The same goes for each β_l'' he means that each of these is a radical expression of the sort referred to in the first sentence of this paragraph.

P.245: ''**PROOF OF THEOREM 4**'' should be **PROOF OF THEOREM 21.4**.

P.245, beginning of proof of Theorem 21.4, through top paragraph of next page: This argument can be stated much more briefly: Suppose we factor t^n-1 into its monic irreducible factors in $\mathbb{Q}[t]$, which by Corollary 3.18 lie in $\mathbb{Z}[t]$. Then each zero of t^n-1 (which by the derivative test for multiple roots has multiplicity 1) is a zero of exactly one of these factors, and that factor is its minimal polynomial. Hence, two zeroes of t^n-1 have the same minimal polynomial if and only if they are zeroes of the same irreducible factor of t^n-1 in $\mathbb{Z}[t]$.

P.246, 4th display, and also 2nd line after that display: The symbols ''(t)'' in the subscripts should be un-subscripted. Thus, at the end of the display, $\bar{m}_{[\varepsilon^p](t)}$ should be $\bar{m}_{[\varepsilon^p]}(t)$, and similarly for the two cases near the beginning of the second line after that.

P.246, last paragraph, beginning ''For each divisor'' through end of proof on next page: This argument is not needed (and also has a gap on the top line of p.247, as well as a confusing change of notation just before that line). Replace this material by the sentence ''Since the primitive nth roots of unity are precisely the elements ε^u for such u, this proves the theorem.''

P.247, bottom of page: Add

> We can now give the
>
> **PROOF OF THEOREM 21.3** Since $\mathbb{Q}(\zeta):\mathbb{Q}$ is a normal extension of $\mathbb{Q}$, every automorphism of $\mathbb{Q}(\theta\zeta)$ over $\mathbb{Q}(\theta)$ carries $\mathbb{Q}(\zeta)$ into itself; so restriction of automorphisms gives a homomorphism ψ: $\Gamma(\mathbb{Q}(\theta\zeta):\mathbb{Q}(\theta)) \to \Gamma(\mathbb{Q}(\zeta):\mathbb{Q})$. We know that the latter group is cyclic of order $p-1$, so it suffices to prove that ψ is an isomorphism. Since $\mathbb{Q}(\theta\zeta) = \mathbb{Q}(\theta)(\zeta)$, every automorphism of this field over $\mathbb{Q}(\theta)$ is determined by what it does to ζ, hence distinct automorphisms act differently on ζ, and so induce distinct automorphisms of $\mathbb{Q}(\zeta)$, showing that ψ is one-to-one. To show that it is onto, it will therefore suffice to show that $\Gamma(\mathbb{Q}(\theta\zeta):\mathbb{Q}(\theta))$ and $\Gamma(\mathbb{Q}(\zeta):\mathbb{Q})$ have the same order.
>
> By Theorem 21.7, for every n the order of $\Gamma(\mathbb{Q}(\zeta_n):\mathbb{Q})$ is the order of $\mathbb{Z}_n^*$, which is the Euler function $\varphi(n)$ (p.42, bottom). Using the Tower Law we conclude that for positive integers r and s we have $|\Gamma(\mathbb{Q}(\zeta_{rs}):\mathbb{Q}(\zeta_s))| = \varphi(rs)/\varphi(s)$. But when r and s are relatively prime, the second display on p.43 tells us that $\varphi(rs) = \varphi(r)\,\varphi(s)$, so
>
> $$|\Gamma(\mathbb{Q}(\zeta_{rs}):\mathbb{Q}\zeta_s)| \;=\; \varphi(rs)/\varphi(s) \;=\; \varphi(r) \;=\; |\Gamma(\mathbb{Q}(\zeta_r):\mathbb{Q})|.$$
>
> Putting $r = p$, $s = p-1$ this becomes the relation needed to complete our proof. []

P.249, Exercises 21.1: ‘‘Section 21.3’’ should be Section 21.4. (This exercise is asking you to verify the first equality of display (21.9) on p.243. Exercises 21.3 and 21.5 likewise ask for verifications of other results asserted on that page.)

Pp.249-250, Exercises 21.6-21.10: The words ‘‘are all 0, ±1’’ should read ‘‘are all contained in $\{0,\pm 1\}$’’. In other words, it is not asserted that in each case, all three values $0, +1, -1$ occur.

P.250, Exercise 25.11: Change the first sentence to ‘‘Assuming the results of Exercises 21.6-21.10, deduce that for all positive integers $n < 105$, all the coefficients of $\Phi_n(t)$ are contained in $\{0,\pm 1\}$.’’

P.251, Definition 2.21. Change this to:

Let G be a permutation group; i.e., a subgroup of the group of all permutations of a set S. We say that G is transitive (*or transitive on S*) *if for all $s,t\in S$ there exists $\gamma\in G$ such that $\gamma(s) = t$.*

If s_0 is any element of S, then to show that a group G of permutations of S is transitive it is enough to show that for all $s\in S$ there exists $\gamma\in G$ such that $\gamma(s_0) = s$, because then given two elements s and t there will exist $\gamma,\delta\in G$ such that $\gamma(s_0) = s$, $\delta(s_0) = t$, whence $(\delta\gamma^{-1})(s) = t$.

P.252, second line of proof of Proposition 22.3: The ‘‘Proposition 4’’ referred to is Proposition 11.4 (on p.126). I’m not sure what ‘‘Theorem 4’’ should be; maybe Theorem 17.4 (p.179).

P.256, statement of Theorem 22.7, point 3: At the beginning, add ‘‘*If $\Delta(f) \neq 0$, then*’’, and after ‘‘*the Galois group of f*’’ add ‘‘*, regarded as a group of permutations of the zeros of f,*’’. In the next-to-last line of the proof of this theorem, both occurrences of ‘‘$\subset$’’ should be ‘‘$\subseteq$’’.

P.257, line before second display: Before the final word ‘‘define’’, insert ‘‘let $x_1,\dots,x_n$ be independent indeterminates, let β be defined as above, and for every $\sigma\in\mathbb{S}_n$’’.

P.257, second display: Although in mathematical notation a subscript as in $\sigma_x(\beta)$ or $\sigma_\alpha(\beta)$ usually means that the expression depends on the value of that subscript, in Stewart’s usage here, it merely signals that in the first case the permutation σ is made to act on the subscripts of the x’s, and in the second that on the subscripts of the α’s.

The maps σ_x $(\sigma\in\mathbb{S}_n)$ give genuine automorphisms of $\Sigma[t,x_1,\dots,x_n]$, which permute the indeterminates. The symbols σ_α do not; they can only be thought of as acting on *expressions* in $x_1,\dots,x_n$ and $\alpha_1,\dots,\alpha_n$, not on their values. (For instance, if $\alpha_1,\dots,\alpha_4$ are the 4th roots of 2, numbered cyclically on the circle around the origin on which they lie, and σ is the permutation (23), then $\alpha_1+\alpha_3 = 0$ in our field, but applying σ_α to the two sides of this equation does not preserve the equality; so σ_α is not a well-defined function on the field. It does, however, have a well-defined value on the expression $\alpha_1+\alpha_3$, namely $\alpha_1+\alpha_2$, and a well-defined value on the expression 0, namely 0.) However, if σ is a permutation that acts on $\alpha_1,\dots,\alpha_n$ by a member of the Galois group of f, then its action on any expression will be equivalent to the action of that automorphism, and in this case it will preserve equalities.

P.257, line before 3rd display: $s\neq$ should be $\sigma\neq$.

P.257, 4th display: Here the subscript i ranges over the set of all n-tuples of nonnegative integers $(i_1,\dots,i_n)$ with the property $i_1+\dots+i_n+j = n$.

P.258, Theorem 22.8: Note that in the statement and proof of this theorem, G and $\mathbf{G}$ have distinct meanings. (Each is a subgroup of $\mathbb{S}_n$; the theorem is aimed at proving them equal.)

P.258, between second and third displays, ‘‘Therefore, H divides Q in $K(x_1,\dots,x_n)[t]$ so that H divides Q in $K[x_1,\dots,x_n][t]$.’’ This uses a version of Gauss’s Lemma for polynomials in several variables (which Stewart has not proved).

P.258, third display: The second equality appears to assume that $\gamma_x\sigma_x(\beta) = \gamma_\alpha^{-1}\sigma_x(\beta)$, but this is incorrect: γ_x and γ_α^{-1} agree on β, but not necessarily on other elements, such as $\sigma(\beta)$. So replace this display, the line right before it, and the line right after it, by:

Conversely, if $\gamma\in G$, then let us apply to the relation $(t-\beta)\,|\,Q_1$ the automorphism γ. Since Q_1 has coefficients in K, we get $(t-\gamma_\alpha(\beta))\,|\,Q_1$. Now

$t-\gamma_\alpha(\beta) = t-\gamma_x^{-1}(\beta) = \gamma_x^{-1}(t-\beta)$, so $\gamma_x^{-1}(t-\beta)\,|\,Q_1$, equivalently, $(t-\beta)\,|\,\gamma_x(Q_1)$. But Q_1 is the unique irreducible factor of Q divisible by $t-\beta$, so $\gamma_x(Q_1) = Q_1$, so $\gamma\in\mathbf{G}$. Thus $G \subseteq \mathbf{G}$.

P.259, Exercise 22.4: In the first phrase, ‘‘δ’’ should be ‘‘δ or $-\delta$’’. In the first entry of the third row of the first determinant, a_1^2 should be α_1^2. In the very first entry of the second determinant, λ_n should be λ_0.

P.260, Exercise 22.6 top line: Change ‘‘is one of’’ to ‘‘is conjugate to one of’’.

P.260, Exercise 22.7, parts (c) and (e): change ‘‘g splits’’ to ‘‘g is reducible’’. (This change could optionally be made in part (d); in that part either version is correct.)

P.262, Definition 23.1, last sentence: After ‘‘*in the obvious way*’’ add ‘‘*in terms of* $\leq$’’.

P.266, 4th line of proof of Theorem 23.12: ‘‘Lemma 6’’ should be Lemma 5.6.

P.267, Exercise 23.8(g): Change ‘‘can be ordered’’ to ‘‘can be given an ordering making it an ordered field’’.

P.269, line 6 of text: Insert a comma before the last word, ‘‘integration’’.

P.270, line before display (24.1): Change this to ‘‘By integrating by parts twice, discarding in each case the function-evaluated-between-limits term, since the power of $1-x^2$ makes the values at both ends zero, rewriting a factor x^2 in one term as $1-(1-x^2)$, and simplifying, we obtain’’.

P.270, line before display (24.2): Before ‘‘[b]y induction’’ add ‘‘After evaluating by hand the cases $n = 0, 1$, we find’’.

P.270, display (24.2) and following line: This would be clearer if P and Q were written $P_n(\alpha)$ and $Q_n(\alpha)$.

P.270, 3rd from last display: Note that the coefficient a^{2n+1} is the numerator of the coefficient α^{2n+1} of (24.2). Stewart has multiplied out by the denominator b^{2n+1}, thus canceling the denominators of the powers of α appearing in P and Q.

P.271, top line, ‘‘where C is a constant’’: This can, clearly, be taken equal to 2.

P.271, line after second display: For ‘‘the superscripts’’ read ‘‘the superscripts on f’’.

P.272, final display: The last ‘)’ on the left-hand side should be ‘]’, and the first ‘(’ on the right-hand side should be ‘[’.