COMMENTS FOR THE INSTRUCTOR USING RUDIN'S *PRINCIPLES OF MATHEMATICAL ANALYSIS*

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In 1993, after teaching from Rudin's *Principles of Mathematical Analysis*, and noting some errata and points that could use clarification, I wrote the author, and convinced him to make some of those changes. Of the changes he did not make, those that I consider important and straightforward are listed, together with additional points discovered when teaching the course in later semesters, on the attached pages of "*ERRATA AND ADDENDA*". Those that he *did* make are listed on the last of those pages, for the benefit of people who have earlier printings. (The easiest way to check whether one does is to look on p.3 of the text at the definition of "order". If the transitivity condition, (ii), has the absurd hypothesis "$x < y$ and $y < x$" instead of "$x < y$ and $y < z$", then the printing is one that needs those additional corrections.) Those sheets might be duplicated and made available to students in courses using the text. I will refer to them below as the errata/addenda.

I give here some comments that are not simple errata, but which you might want to take into account in teaching from Rudin. Comments and corrections concerning these pages of comments and corrections are welcomed! The course I have taught only covers chapters 1-7 of the text, so my notes apply to these; but I would consider adding important notes relevant to the later chapters.

I have also put together an 89-page packet supplementing the exercises in chapters 1-7 of Rudin. It consists mainly of new exercises (ranging from some very challenging ones to elementary true-false questions with answers on the next page, which students are advised to use to check their understanding of basic results and definitions), and also contains information on which section of each chapter Rudin's various exercises for the chapter go with, estimates of the difficulty and mutual dependence of those exercises, and clarifications of some of them. That packet is located at

`http://math.berkeley.edu/~gbergman/ug.hndts/m104_Rudin_exs.pdf`

I refer to it in places below as "the exercise-packet".

The present pages can be found online at

`http://math.berkeley.edu/~gbergman/ug.hndts/m104_Rudin_notes.pdf`

If you're using them significantly later than the last date shown at the top of this page, you might check there to see whether there is a later version.

Here, now, are the comments on Rudin:

P.2, formula (3): Students are baffled by this "rabbit-out-of-a-hat" definition. One should motivate it, or tell the class, "Take it for granted, without worrying about where it comes from" – or something!

I generally partially motivate it, noting that if $p^2 < 2$ we want to increase p slightly, while if $p^2 > 2$ we want to decrease it, so the amount we should change it by should be obtained from $p^2 - 2$. A denominator is needed to prevent overshooting, especially when p is large, so we use one that grows with p, but I said the actual choice of denominator $p + 2$ can be regarded as the result of trial and error. For a lengthier but more satisfying motivation, see the exercise packet, exercise 1.1:1, and for a shorter way of getting the result that A has no greatest and B no least element, exercise 1.1:2.

P.6, Proposition 1.14: Statement (e) in the errata/addenda is used implicitly by Rudin at later points.

P.16, Theorem 1.37: Statement (g) in the errata/addenda complements statement (d); the notes in the errata/addenda to pp.113 and 135 show its usefulness.

P.24, Definition 2.2: One may wish to point out that the use of $f(E)$ for $\{f(x) \mid x \in E\}$ is ambiguous in situations where the same set E can occur both as a *member* and a *subset* of the given set A – but that such situations will not occur in this course.

P.26, Definition 2.7: I tell my class that $\{x_n\}$ for a sequence is bad notation, and that I will instead use (x_n) or $(x_n)_{n \in J}$.

P.32, Definition 2.18: One should point out that most of these definitions are *relative*: If X is a subspace of a metric space Y, then whether a given subset of X is a neighborhood of a point x, whether a given point is isolated, etc., depend on whether we are regarding these within X or within Y. These distinctions are needed in the subsequent reading.

P.59, line before third display: Rudin describes the symbols $a_1 + a_2 + a_3 + \ldots$ and $\Sigma\, a_n$ as abbreviations for the sequence $\{s_n\}$ of partial sums. I have indicated in the errata/addenda that they denote, rather, the limit, if any, of that sequence.

I think Rudin's reason for his definition is that he wants to speak of $\Sigma\, a_n$ as being "convergent" or "divergent", and this is applicable to the sequence, but not, logically, to its limit. I have thought about this kind of question. It is true that when one talks about $\Sigma\, a_n$, one is sometimes talking about the limit-value, and other times about the process by which we get it. But if we define the expression to mean the sequence of partial sums, this makes our way of interpreting *equations* in such expressions totally unnatural, while if we define it to mean the limit-value (assuming this exists), then the equations are correct, and statements about, say, the nature of the convergence are simply examples of the common phenomenon where, in talking about a mathematical entity, if we want to refer to properties of the process by which it was constructed, we often abuse language by referring to these as properties of the entity. (Other familiar entities that we often treat in this way are limits themselves, and determinants.)

If we followed Rudin's definition, then statements like "$\Sigma\, a_n$ converges absolutely" or "$\Sigma\, b_n$ is a rearrangement of $\Sigma\, a_n$" still could not be defined in any natural way, because these refer back one step further in the process of constructing the sum than the sequence of partial sums.

P.63, last 7 lines of middle paragraph, beginning "One might thus be led": When I first read this, I took "boundary" to mean something like the line one draws around a set in a Venn diagram, and couldn't see why he said one didn't exist. I finally realized what he meant; see exercise 3.7:1 in the exercise packet.

P.68, Remark 3.36: I amplify this point by writing on the board "The ratio test is unfair!", and compare it to the situation where a person is judged by how much he or she has "improved", and is thus judged better if he or she had given a poorer performance earlier. Similarly, the ratio test measures each term *relative* to the one that precedes, and hence makes a series look "bigger" (less likely to converge) if smaller terms are mixed in. E.g., if one mixes terms of 3^{-n} in with those of 2^{-n}, it looks to the ratio test as though we may have a divergent series. The root test, in contrast, measures how small the terms are really getting.

P.76, Theorem 3.54: This result suggests the following question. Suppose $\Sigma\, a_n$ is a conditionally convergent series of vectors in R^k; what can the set of sums of its convergent rearrangements look like? It is not hard to show that any translate of any vector subspace of R^k can occur as this set, for appropriate choice of the series. The converse is not so obvious, but it is true; cf. Peter Rosenthal, *The remarkable theorem of Lévy and Steinitz,* Amer. Math. Monthly **94** (1987) 342-351. MR 88d:40005.

P.90, proof of Theorem 4.17: If instead of Theorem 4.8 one uses the Corollary to that theorem, the proof becomes shorter and more direct.

P.110, Taylor's Theorem: It is worth pointing out that the usefulness of this theorem is for functions f whose sequence of derivatives $f^{(n)}(x)$ can be shown not to grow "too fast" as functions of n, so that for appropriate t, the error term in (24) goes to zero as $n \to \infty$. That this fails for $f(x) = e^{-x^{-2}}$ is shown in exercise 5.6:1 in the exercise-packet, and, later, by Rudin in Exercise 1 of Chapter 8 (p.196).

P.112, top half: As background, one should have done more of the theory of limits of functions for the vector-valued case, and defined $\mathbf{a}/b$ when $\mathbf{a}$ is a vector and b a scalar.

P.127, display (18): If you replace δ^2 on the right with $\delta\varepsilon$, then you can drop the assumption $\delta < \varepsilon$ four lines earlier. The δ^2 in (19) will then also change to $\delta\varepsilon$, and on the next line, $< \delta$ will become $< \varepsilon$, which is what is actually used in the line that follows.

P.157, Theorem 7.24: It would be neater to say that a uniformly convergent sequence of uniformly continuous functions on *any* metric space X is equicontinuous.

P.159, first two lines of proof: Students have a lot of trouble with this "without loss of generality" assertion. I explain to them that mathematicians say "Without loss of generality we may assume P holds" to mean "If we knew the result in the case where P holds, we could deduce the general case, so let's assume we are in that case". Rudin makes two such reductions on these two lines, and students unfamiliar with such arguments should see both carefully justified.

P.160, first paragraph. An easier way to see the inequality in question is to note that for any nonnegative α and β, we have $(1-\alpha)(1-\beta) \geq 1-\alpha-\beta$, and proceed by induction to a product of n factors.

P.163, last two lines, and p.164, first 3 lines: A clearer notation would be to write $h_{x,y}$ instead of h_y.

P.165, last sentence of middle paragraph of proof of Theorem 7.33: Easier: instead of multiplying g by a cunningly chosen scalar λ, multiply it by $\overline{g} \in \mathcal{A}$.

ERRATA AND ADDENDA TO CHAPTERS 1-7
OF RUDIN'S *PRINCIPLES OF MATHEMATICAL ANALYSIS*,
3rd Edition (noted as of December, 2006)

For additional errata to **earlier printings**, see last page of these sheets.

⇒ Note: If you don't want to write corrections into your text, you might put them on PostIts (or slivers of paper cut from PostIts) and insert these at the page in question. ⇐

P.4, 6th line of Example 1.9: "lasgest" should be "largest"

P.4, 3rd line of Definition 1.10: A clearer statement would be, "Every subset $E \subset S$ which is nonempty and bounded above has a supremum $\sup E$ in S."

P.5, last 5 lines of proof of Theorem 1.11: Change these lines to:

> If α were not a lower bound of B, there would be some $x \in B$ satisfying $x < \alpha$. This x would be an upper bound of L (by the preceding paragraph), contradicting our assumption that α is the *least* upper bound of L. So α is a lower bound of B. Now if y is any lower bound of B, then $y \in L$, so $y \leq \sup L = \alpha$; this shows that α is the *greatest* lower bound of B.

P.6, Proposition 1.14: Add

(*e*) $-(x+y) = (-x)+(-y)$.

Can you see how to prove this? I will either discuss it in class, or make it an exercise.

P.12, lines 8-11, definitions of operations on the extended real numbers: Rudin should have noted the convention that $x + (+\infty)$ and $x + (-\infty)$ may be abbreviated $x + \infty$ and $x - \infty$ respectively, and mentioned that addition and multiplication are understood to be commutative on the extended reals, so that the definitions he gives also imply further cases like $+\infty + x = +\infty$. Finally, the three equations in (a), instead of having the common condition "If x is real", should be preceded by the respective conditions, "If x is real or $+\infty$", "If x is real or $-\infty$", and only in the last case simply "If x is real".

P.16, Theorem 1.37: Add one more part:

(*g*) *Assuming* $k > 0$, *there exists a vector* $\mathbf{u}$ *with* $|\mathbf{u}| = 1$ *such that* $\mathbf{u} \cdot \mathbf{x} = |\mathbf{x}|$.

Proof: If $\mathbf{x} \neq 0$ let $\mathbf{u} = |\mathbf{x}|^{-1}\mathbf{x}$; if $\mathbf{x} = \mathbf{0}$ let $\mathbf{u}$ be any vector with $|\mathbf{u}| = 1$.

P.19, middle: The author refers to the archimedean property of Q. This is *not* a consequence of Theorem 1.20 (a); that would be circular reasoning. Rather, it is an elementary property of Q : Given $x, y \in Q$ with $x > 0$, we need to find an $n > y/x$. If $y/x < 0$, take $n = 1$; otherwise, write y/x as a fraction with positive denominator, and take for n any integer greater than its numerator.

P.29, second line after display (17): Change "Hence there is a subset" to "Hence we cannot say that the map sending the natural number n to the nth term of this sequence is a one-to-one correspondence; but clearly there is a *subset*".

P.31, Definition 2.17: In defining "the *segment* (a,b)", "the *interval* $[a,b]$", etc., Rudin doesn't say whether a and b are real numbers or extended real numbers. In at least one place, namely exercise 29 to this chapter (p.45), one must understand them to be extended real numbers, so that, for instance, R can be considered as the segment $(-\infty, \infty)$. (I haven't had time to examine whether this interpretation is consistent with all Rudin's uses of these terms.) In any case, the definition of "segment" should contain the assumption "$a < b$" (otherwise the empty set would be a segment, which is not desired), while the definition of "interval" should contain the assumption "$a \leq b$". The possibility of equality is assumed in the display in the proof of Theorem 2.38; again I'm not sure whether Rudin is consistent about this.

P.35, Proof of Theorem 2.27 (a): Change this to, "We must show that any limit point p of $\overline{E}$ lies in $\overline{E}$. Now any neighborhood N of p contains some $q \in \overline{E}$. Since N is open, N contains some neighborhood M of q, and since $q \in \overline{E}$, M contains some $r \in E$. Thus $r \in M \subseteq N$, so every neighborhood N of p contains a point $r \in E$, so $p \in \overline{E}$."

P.36: After finishing the section of metric spaces, you might find the following discussion enlightening; but it is not required reading.

What is topology? Chapter 2 of Rudin is entitled "Basic Topology", but the chapter is about metric spaces, and the word "topology" does not appear in that chapter, nor in the index. What does it refer to?

Topology is a field of mathematics that *includes* the study of metric spaces as a special case. The key to the connection between metric spaces and the more general concept of a *topological space* is Theorem 2.24, parts (a) and (c) (p.34), which show that if we write T for the set of all open sets in a metric space, then the union of any family of members of T, and the intersection of any *finite* family of members of T, are also open sets. Families of sets with these properties come up in other contexts as well; so one makes

Definition. A *topological space* X means a pair $(X,\ T)$, where X is a set, and T is a set of subsets of X which satisfies

(i) For any collection $\{G_\alpha\}$ with all $G_\alpha \in T$, one has $\bigcup_\alpha G_\alpha \in T$.

(ii) For any finite collection $\{G_\alpha\}$ with all $G_\alpha \in T$, one has $\bigcap_\alpha G_\alpha \in T$.

(iii) $\emptyset \in T$ and $X \in T$.

T is called a *topology* on X, and the sets in T are called the *open sets* of X with respect to this topology.

When T has been specified, and there is no danger of ambiguity, one simply speaks of "the topological space X" and "open sets of X".

(*Remark*: Condition (iii) can be omitted from this definition if one interprets conditions (i) and (ii) appropriately, since $\emptyset$ can be regarded as the union of the *empty* family of members of T, and if one interprets $\bigcap$ to refer to intersection as subsets of X, then X can likewise be regarded as the intersection of the empty family of members of T.)

Most of the concepts developed in Chapter 2 can be expressed in terms of open sets, hence also make sense in a general topological space. For instance, a *closed set* G can be defined as a set whose complement G^c is open. (Under this definition, parts (c) and (d) of Theorem 2.24 clearly hold in any topological space.) A *limit point* of E can be defined as a point $p \in X$ such that every open subset of X which contains p contains a point of E other than p (cf. exercise 2.2:4 in the exercise packet). In terms of limit points, one can define *isolated point* and *perfect set*. (One can also check that Rudin's definition of "closed set" in terms of "limit point" yields, in this context, the class of sets we just defined to be closed.) One can define the *interior* E° of a subset E to be the union of all open sets contained in E, and an *interior point* to be a point of E°. Rudin's definition of *compact set* (given at the beginning of the next section) will be stated in terms of open sets, so it, too, makes sense in this context.

Of the main concepts defined for general metric spaces in Chapter 2, there are two that don't have analogs in the general theory of topological spaces: those of "neighborhood" and of "bounded subset"; these are among the features that distinguish the theory of metric spaces from the general theory of topological spaces. (Actually, topologists define a "neighborhood" of a point $p \in X$ to be any subset $E \subset X$ having p in its interior. In current usage, what Rudin calls a "neighborhood" is called an "open ball", so in modern language, it is the concept of "open ball" that is meaningful for metric spaces but not for general topological spaces.)

Why is it useful to study general topological spaces, and not just metric spaces? There are two reasons. One is that there are examples of topological spaces that don't arise from a metric. For instance, if X is any infinite set, one can take T to consist of all subsets $G \subset X$ such that either $G = \emptyset$ or $X - G$ is finite; this is a topology on X having properties that a topology arising from a metric can never have.

The other reason is that different metrics can correspond to the same topology, and it is sometimes important to realize that certain spaces are "topologically the same" even though they look different as metric spaces. As a trivial example, if d is any metric on a set X, then the metric d' given by $d'(x, y) = d(x, y)/2$ determines the same topology as d. For a less trivial example, let d be the ordinary metric on the segment $(-1, 1) \subset R$, and let d'' be the metric defined by $d''(x, y) = |(\tan \pi x/2) - (\tan \pi y/2)|$. Since the function $\tan \pi x/2$ "stretches" the segment $(-1, 1)$ to fill up the whole real line, d'' can be thought of as the metric on $(-1, 1)$ induced by the ordinary metric on the "stretched" segment, the whole line. It is easy to show that the open subsets are the same under both metrics, namely the sets that can be written as unions of open intervals (in Rudin's language, as unions of segments); so we are talking about the same topology on our set $(-1, 1)$; but under one metric, the set is bounded, and under the other, unbounded. (Similar "stretching" can change other commonplace shapes into very "different-looking" ones; in particular, there is a "stretching" that leads to the familiar quip that a topologist is a person who doesn't know the difference between a donut and a coffee-cup.)

A well-written standard introduction to topology is *General Topology* by John Kelley, Van Nostrand, 1955.

P.36, end of Definition 2.31: Add, "If $\{G_\alpha\}$ is an open cover of E, then by a subcover we mean a subset of $\{G_\alpha\}$ which is also a cover of E."

P.41, next-to-last paragraph of proof of Theorem 2.43: Replace this with the following three paragraphs, leaving the rest of the proof unchanged:

> Starting with V_1, we shall construct recursively a sequence of neighborhoods V_n with the following properties: (i$_n$) $V_n \cap P$ is not empty, (ii$_n$) If $n > 1$, then $\overline{V_n} \subset V_{n-1}$, (iii$_n$) If $n > 1$, then $\mathbf{x}_{n-1} \notin \overline{V_n}$.
>
> Suppose inductively that V_n has been constructed. We claim that it contains a point $\mathbf{y} \in P$ other than $\mathbf{x}_n$. Indeed, by (i$_n$) it contains some point $\mathbf{z} \in P$, and if $\mathbf{z} \neq \mathbf{x}_n$ we are done. If $\mathbf{z} = \mathbf{x}_n$, note that since V_n is open, it contains some neighborhood U of $\mathbf{z}$, and because P is perfect, U contains some point $\mathbf{y} \in P$ other than $\mathbf{z}$. So let $\mathbf{y}$ be so chosen.
>
> Now, because V_n is open, it also contains a neighborhood of $\mathbf{y}$, say of radius r. Let us take for V_{n+1} any neighborhood of $\mathbf{y}$ whose radius is both $< r$ and $< d(\mathbf{x}_n, \mathbf{y})$. From the first of these conditions one can deduce that (ii$_{n+1}$) holds and from the second that (iii$_{n+1}$) holds. Finally, the fact that $\mathbf{y} \in V_{n+1}$ gives (i$_{n+1}$), as required.

P.42, top paragraph: An easier way to see that P (the Cantor set) contains no segment is to note that E_n contains no segment of length $> 3^{-n}$. (If you keep Rudin's argument, change "positive" to "nonnegative" on line 3.)

P.45, exercises 22, 23, 25, 26 and 28: Some minor corrections to these are noted in the exercise packet.

P.48, Theorem 3.2: Add: (e) *If* $\lim_{n\to\infty} p_n = p$, *and* $p_n \in E$ *for all* n, *then* $p \in \overline{E}$.

P.49, Theorem 3.3 (b): *any number* should be *any complex number*.

P.49, Theorem 3.3: Add part (e): *If* $s_n \leq t_n$ *for all* n, *then* $\lim_{n\to\infty} s_n \leq \lim_{n\to\infty} t_n$. This is proved by noting that $\lim_{n\to\infty}(t_n - s_n)$ is a limit of elements of $[0, \infty)$, hence belongs to that closed set by Theorem 3.2 (e) .

P.53, Theorem 3.10: The sets E and K_n must be assumed nonempty.

P.54, two lines before Def. 3.12: "(Theorem 2.41)" should be "(Theorem 2.41 and Theorem 3.10 (a))".

P.54, two lines after Definition 3.12: Change "Theorem 3.11 implies also" to "Definition 3.12 implies".

P.56 Theorem 3.17 (b): Change "If $x > s^*$" to "*For every real number* $x > s^*$". (In particular, x does not stand for a member of E, as it does earlier on the page.)

P.56 6th from last line: Change "In that case, there is a number $y \in E$ such that" to "These form a subsequence of $\{s_n\}$ consisting of numbers $\geq x$. Some subsequence of that subsequence approaches a value y in the extended real numbers, and this y belongs to E and satisfies".

P.57, line 5: (-1^n) should be $(-1)^n$.

P.59, line after 2nd display: Change "For $\{s_n\}$" to "For the limit of the sequence $\{s_n\}$, if this exists,".

P.63, start of line 3: Change "$\{1/n \log n\}$ decreases" to "for $p \geq 0$, $\{1/(n(\log n)^p)\}$ decreases".

P.66, First line of Theorem 3.34: Change "*The series* $\Sigma\, a_n$" to "*A series* $\Sigma\, a_n$ *of nonzero terms*".

P.67, third line: $n \geq N$ should be $n > N$.

P.67, middle bank of equations, third equation (the one ending with $1/\sqrt{2}$) : The superscript in the $\sqrt{\ }$ should be ${}^{2n-1}$ rather than ^{2n}.

P.70, Theorem 3.42: Change the first word, "*Suppose*", to "*Let* $\{a_n\}$, $\{b_n\}$, $\{A_n\}$ *be as in Theorem 3.41, and suppose*".

P.73, 3rd line of Example 3.49: Change the initial word "and" to "where c_n is defined as in Definition 3.48, and if".

P.77, 3rd line after (25): after "$\alpha_n < \beta_n$," add "$\alpha_{n-1} < \beta_n$,".

P.84, first line: Change "if there is a point $q \in Y$" to "if $q \in Y$ is a point".

P.84, first line of proof of Theorem 4.2: Change "Choose" to "Consider any".

P.94, 2nd display: Change "$= \lim_{t\to x} f(t)$" to "and when this holds, $\lim_{t\to x} f(t)$ is their common value".

P.98, line 7: Change "is not empty" to "has points other than x".

P.98, first line of Theorem 4.34: Change "be defined" to "be real functions defined".

P.104, third line above Theorem 5.3: Change "isolated" to "single".

P.105, proof of Theorem 5.5: In the line after (5), before "Let" add: "We also define $u(x)=0$ and $v(y)=0$; thus u is continuous at x, and v at y." Then change the last two lines of the proof to "By Theorem 4.7 the right-hand side of (6) is continuous at $t=x$, where it has value $g'(y)f'(x)$, hence the left-hand side approaches this value as $t\to x$, which gives (3)."

(Rudin's proof skirts the point that makes proving the Chain Rule difficult: that $f(t)$ may take on the value $f(x)$ infinitely often in the neighborhood of x.)

P.108, change the title **THE CONTINUITY OF DERIVATIVES** to **A RESTRICTION ON DISCONTINUITIES OF DERIVATIVES**.

P.109, display (13): Between "A" and "*as*" insert "(*a real number or* $\pm\infty$)".

P.109, add as a footnote to (18): "Note that by assumption, g' is nowhere 0 on (a,b). Hence by the Mean Value Theorem, $g(x)-g(y)$ is nonzero for distinct $x,y\in(a,b)$."

P.113, Theorem 5.19: For a simpler proof, use Theorem 1.37 (g) (given in the note to p.16 above) to choose $\mathbf{u}$ so that $\mathbf{u}\cdot(\mathbf{f}(b)-\mathbf{f}(a))=|\mathbf{f}(b)-\mathbf{f}(a)|$, and apply the Mean Value Theorem to $\mathbf{u}\cdot f(t)$.

P.115, Exercise 13, display defining $f(x)$: x^a should be $|x|^a$.

P.118, first display: *Change* "$\begin{smallmatrix}1\\2\end{smallmatrix}$" to "$\frac{1}{2}$".

P.123, Definition 6.3: All these partitions should be understood to be of a fixed interval $[a,b]$.

P.126 (17): $i-$ should be $i=$.

P.128 Theorem 6.12: In (a), in the 3rd and 5th lines, change f (without subscript) to f_1 (three occurrences). In (b), add the assumption that $f_1,f_2\in\mathcal{R}(\alpha)$.

P.129 line after (21): One must assume that the second statement of part (a) (about the integral of cf) has been proved before the first statement, and use that with $c=-1$.

P.130 first line of Theorem 6.16: "for $1,2,3,\ldots$" should be "for $n=1,2,3,\ldots$".

P.135: The inequality (40) of Theorem 6.25, like Theorem 5.19 (note to p.113 above) can be proved more easily using Theorem 1.37 (g). Do you see how? (Also, in the proof as Rudin gives it, in the next-to-last display on this page, y_i^2 should be y_j^2, and later in that line, y_J should be y_j.)

P.141, 4th line from bottom: Change $[0,2\pi]$ to $(0,2\pi]$.

P.150, 4th line of proof of Theorem 7.13, "(Theorem 4.8)": Better, the Corollary to that theorem.

P.155, Example 7.20: Rudin asserts nonexistence of a pointwise convergent subsequence of $(\sin nx)$, calling on a result in Chapter 11. An elegant direct proof is given in exercise 7.6:2 in the exercise packet.

P.158, first line of proof of statement (b): Change "a countable" to "an at most countable".

P.161, last line: change "Theorem 2.27" to the more precise "Theorem 2.27 (a)".

P.165, last word of Theorem 7.33: After "*dense*" add "*in*".

P.166, line following first display: After "converges" add "on $[0,1]$".

(I've only taught through chapter 7, but the next two corrections were sent to me by a student who read through chapter 8:)

P.179, display (28): under both limit-signs, "$h=0$" should be "$h\to 0$".

P.185, line 3: change "$>\mu$" to "$>\mu+1$". (Needed to be sure $\mu=\inf|P(z)|$ implies $\mu=\inf_{|z|<R_0}|P(z)|$.)

P.336, line 3: Change "Pure Mathematics" to "A Course of Pure Mathematics".

ADDITIONAL CORRECTIONS TO MAKE IF YOU HAVE AN EARLIER PRINTING OF RUDIN'S *PRINCIPLES OF MATHEMATICAL ANALYSIS*

(To tell whether you do, just see whether the first of the items below has been fixed in your copy.)

P.3, Definition 1.5, condition (ii): Change "and $y<x$" to "and $y<z$".

P.10, 2nd line of Theorem 1.21: Change "*one real* y" to "*one positive real* y".

P.10, 5th line of **Proof**: Change "$t^n<t$" to "$t^n\le t$"; and two lines later change "$t^n>t$" to "$t^n\ge t$".

P.17, Step 1, item (I): Change the comma to the word "and".

P.32, 4th line: After "$d(p,q)<r$", add "for some $r>0$".

P.33, under Examples 2.21, description of (c): Change "finite set" to "finite nonempty set".

P.48, 3rd line of Theorem 3.2: Change "all but finitely many of the terms of $\{p_n\}$" to "p_n for all but finitely many n".

P.52, 3rd line from bottom: Change "subset" to "nonempty subset".

P.54, 5 lines above Definition 3.12: Change "$\{x_n\}$" (with italic x) to "$\{\mathbf{x}_n\}$" (with boldface $\mathbf{x}$), as on next line.

P.57, Example 3.18 (b): Change "$(-1^n$" to "$(-1)^n$".

P.66, Theorem 3.34 (b): Change "for n" to "for all n".

P.67, center of page, last of the four displayed lines beginning "lim sup": Change $(\frac{3}{2})^n$ to $\frac{1}{2}(\frac{3}{2})^n$.

P.72, line 2: Change equation under second $\sum$ from "$n = k$" to "$k = n$" (as under first $\sum$).

P.75, 3rd line from bottom: Change "J to J" to "J onto J".

P.82, beginning of 4th line: Change "and bounded" to "nonempty and bounded".

P.84, five lines above Theorem 4.2: Change "appropriate norms" to "norms of differences".

P.98, first line of Definition 4.33 and first line of Theorem 4.34: Change "E" to "$E \subset R$".

P.123, line following first display: Change "are the same" to "mean the same thing".

P.158, 5th line after display (44): Change "$|f(x)| <$" to "$|f_n(x)| <$".

P.162, line 2: Change "distincts point" to "distinct points".