Comments, corrections, and related references welcomed, as always!

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SOME QUESTIONS FOR POSSIBLE SUBMISSION TO THE NEXT KOUROVKA NOTEBOOK

GEORGE M. BERGMAN

Background.
I had meant to submit a few questions to the latest edition of the Kourovka Notebook [1], but I was busy, and it came out before I had done so. So I have decided to gather ahead of time those I will submit to the next.

Looking over past papers, I found a number of group-theoretic questions that I had raised. I list most of these below, together with a few new ones. (I am temporarily holding back on one, which a student who had to drop out of school still hopes to work on before the next Kourovka Notebook comes out.) I omit a few that seemed too dependent on the contexts of the papers in which they appeared, and, of course, those already in the Kourovka Notebook.

Let me know if you have any comments – in particular if, to your knowledge, any of these questions have been asked before, or if you know or can see any answers.

Questions.

Q 1. For G a group and $n \geq 2$, let us say G has the *unique n-fold product* property, which we will write "u.-n-p.", if for every n-tuple of finite nonempty subsets $A_1, \dots, A_n \subseteq G$, there exists $g \in G$ which can be written in one and only one way as $g = a_1 \dots a_n$ with $a_i \in A_i$. It is easy to see (since some of the A_i can be $\{1\}$) that for $n \geq m$ we have u.-n-p. $\Longrightarrow$ u.-m-p..

Are the conditions u.-n-p. all equivalent; i.e., equivalent to u.-2-p., the usual unique product condition (often denoted "u.p.")?

It is known that a *diffuse* group (definition recalled below in Q 2, second sentence) satisfies u.p.; in fact, it is easily shown that such a group satisfies u.-n-p. for all n. Kionke and Raimbault [19, Question 1] ask whether there is a group with u.p. which is *not* diffuse. If the answer is negative, this would thus imply a positive answer to the present question. If, on the other hand, the answer is positive, then one may ask whether a group having u.-n-p. for *all* $n \geq 2$ must be diffuse.

Q 2. If S is a subset of a group G, then an element $s \in S$ is called an *extremal point* of S if for every $g \in G - \{1\}$, either $sg^{-1} \notin S$ or $sg \notin S$. A group G is said to be *diffuse* if every finite nonempty subset of G has an extremal point. In [20, Proposition 6.2], this condition on G is shown equivalent (inter alia) to the condition called LIO (locally invariant order), namely that G admits a set-theoretic total ordering $\geq$ such that for all $s \in G$ and $g \in G - \{0\}$, at least one of $sg^{-1} > s$ or $sg > s$ holds.

Let us say that G has SLIO (strongly locally invariant order) if it admits an ordering such that for all such s and g, *exactly* one of $sg^{-1} > s$ or $sg > s$ holds. (In other words, every sequence of the form $(sg^i)_{i \in \mathbb{Z}}$ is monotone increasing or monotone decreasing.)

It is not hard to show that

$$\text{right-orderable} \implies \text{SLIO} \implies \text{LIO (equivalently, diffuse).}$$

The composite implication is known not to be reversible [19, Appendix B].

But is one or the other of the above two implications reversible?

This note can be viewed at http://arxiv.org/abs/1904.04298 and at http://math.berkeley.edu/~gbergman/papers/unpub/418.preKourovka.pdf . The latter file may be updated more often than the former.

Q 3. (i) [24], [2, Definition 1 and Question 2] Let us call group G *resistant* if for every field k, and every element $r = \sum_i c_i\, g_i$ of the group algebra $k\,G$ whose support, $\mathrm{supp}(r) = \{g_i \mid c_i \neq 0\}$, has cardinality > 1, the 2-sided ideal of $k\,G$ generated by r is proper.

Are all free groups resistant?

The above question is the motivation of [2], where many sorts of groups are proved non-resistant; e.g., any group containing a nonabelian solvable subgroup. Six more questions are asked there, of which I record two here. (The numbering shown for these might change in subsequent revisions of [2].)

(ii) [2, Question 22] Is the class of resistant groups closed under taking direct products?

(iii) [2, Definition 24 and Question 25] (Possible Freiheitssatz for group algebras) For F the free group on generators $(x_i)_{i\in I}$, and k a field, let us call an element $r \in k\,F$ *strongly reduced* if, when the elements of $\mathrm{supp}(r)$ are written in normal form, (a) there is no symbol $x_i^{\pm 1}$ with which all these elements begin, (b) there is no symbol $x_i^{\pm 1}$ with which all these elements end, and (c) if $1 \in \mathrm{supp}(r)$ (so that the preceding two conditions hold trivially) there is no symbol $x_i^{\pm 1}$ such that all elements of $\mathrm{supp}(r)$ other than 1 both begin with $x_i^{\pm 1}$ and end with the inverse symbol, $x_i^{\mp 1}$.

For any $r \in k\,F$, we shall say that a generator x_{i_0} is "involved in" r if x_{i_0} or $x_{i_0}^{-1}$ occurs anywhere in the normal form of any of the elements of $\mathrm{supp}(r)$.

Suppose I is the 2-sided ideal of $k\,F$ generated by a strongly reduced element r. If $F' \subset F$ is the subgroup generated by $(x_i)_{i\in I-i_0}$, where x_{i_0} is a generator involved in r, must the composite map $k\,F' \hookrightarrow k\,F \to (k\,F)/I$ be an embedding? (A positive answer would imply a positive answer to (i).)

Q 4. [15, Question 13] Let $(G, \leq)$ be a right-ordered group, and k a field. Thus, the set of formal infinite sums $\sum_{g\in G} a_g g$ with coefficients $a_g \in k$ whose support $\{g \in G \mid a_g \neq 0\}$ is well-ordered forms a right module $k((G))$ over the group algebra kG, though this will not in general have a natural structure of ring, or of left kG-module.

Dubrovin [17] showed that every nonzero element of kG acts *invertibly* on the module $k((G))$. We ask:

For each $x_1,\, x_2,\, y_1,\, y_2 \in kG - \{0\}$, will the right action of $y_1 y_2^{-1} - x_1 x_2^{-1}$ on $k((G))$ either be invertible or zero? (Here y_2^{-1} and x_2^{-1} denote the inverses of the *actions* of y_2 and x_2.)

The above is easily seen to be equivalent to the same question for the actions of $x_1^{-1}y_1 - x_2^{-1}y_2$, of $y_1 - x_1 x_2^{-1} y_2$, and of $1 - x_1 x_2^{-1} y_2\, y_1^{-1}$.

An affirmative answer to this question is equivalent to a matrix-theoretic property asked for in [15, Question 12]. Such a result would be the "next step", after Dubrovin's result, toward proving that kG is embeddable in a division ring, and indeed generates a division ring of operators on $k((G))$. A countable chain of conditions, having Dubrovin's result as its first step and the property asked for above as its second, and which all together are equivalent to generating a division ring of operators, is developed in [15, §10].

For simplicity I am posing this question for k a field rather than a general division ring. If a positive answer is obtained, one should, of course, see whether commutativity of k is really needed, and whether the result can be extended to group rings twisted by actions of G on k.

The next four questions, Q 5-8, taken from [14], concern homomorphic images of ultra-products of groups (with a slight exception in Q 8(ii)).

Q 5. (i) [14, Question 16, p.468] If $\mathcal{U}$, $\mathcal{U}'$ are nonprincipal ultrafilters on ω (the set of natural numbers), can every group B which can be written as a homomorphic image of an ultraproduct of groups with respect to $\mathcal{U}$ also be written as a homomorphic image of an ultraproduct of groups with respect to $\mathcal{U}'$?

(ii) [14, Question 17, p.468] If the answer to (i) is negative, is it at least true that for any two nonprincipal ultrafilters $\mathcal{U}$ and $\mathcal{U}'$ on ω, there exists a nonprincipal ultrafilter $\mathcal{U}''$ on ω such that every group which can be written as a homomorphic image of an ultraproduct of groups with respect to $\mathcal{U}$ *or* with respect to $\mathcal{U}'$ can be written as a homomorphic image of an ultraproduct with respect to $\mathcal{U}''$?

(A positive answer to (ii) would imply that the class of groups which can be written as homomorphic images of nonprincipal countable ultraproducts of groups is closed under finite direct products.)

Q6. [14, Question 18, p.468] Can either of the following groups be written as a homomorphic image of a nonprincipal ultraproduct of a countable family of groups?

(i) An infinite finitely generated Burnside group?

(ii) The group of those permutations of an infinite set that move only finitely many elements?

([14, Corollary 14] shows that no group of permutations containing an element with exactly one infinite orbit can be written as an image of such an ultraproduct, and the next paragraph of [14] sketches how this class of examples can be generalized.)

Q 7. [14, Question 19] If $\mathcal{U}$ is an ultrafilter on ω, and B is a group such that every $b \in B$ lies in a homomorphic image within B of $\mathbb{Z}^\omega/\mathcal{U}$, must B be a homomorphic image of an ultraproduct group $(\prod_{i\in\omega} G_i)\,/\,\mathcal{U}$?

(A positive answer would be the converse to [14, Lemma 12]. Such a result seems highly unlikely. It would, in any case, imply that every torsion group was such a homomorphic image for every $\mathcal{U}$, and so would give positive answers to both parts of Q 6 above.)

Q 8. (i) [14, Question 35, p.483] If an *abelian* group B can be written as a homomorphic image of a nonprincipal countable ultraproduct of *not necessarily* abelian groups G_i, must it be a homomorphic image of a nonprincipal countable ultraproduct of *abelian* groups?

(ii) (Not from any of my papers.) If an abelian group A can be written as a homomorphic image of a *direct product* of an infinite family of not necessarily abelian *finite* groups, $A = h(\prod_{i\in I} G_i)$, can A be written as a homomorphic image of a direct product of finite *abelian* groups, $A = h'(\prod_{j\in J} A_j)$?

(To see that neither question is trivial, choose for each $n > 0$ a finite group G_n which is perfect, but has elements which cannot be written as products of fewer than n commutators. Then both the direct product of the G_n and any nonprincipal ultraproduct of those groups will have elements which are *not* products of commutators; hence its abelianization A will be nontrivial. There is no evident family of abelian groups from which to obtain A as an image of a nonprincipal ultraproduct, nor a family of finite abelian groups from which to obtain A as an image the direct product.)

One can ask similar questions with "{abelian groups} $\subset$ {groups}" replaced by any term and later term in,

$$\{\text{abelian groups}\} \subset \{\text{groups}\} \subset \{\text{monoids}\} \subset \{\text{magmas}\},$$

and similarly for various other classes of mathematical structures.

Q 9. [13, Question 11, p.80] It is shown in [13, Theorem 9 and Corollary 10, p.79] that the group $\mathbb{Z}^\omega$ has a subgroup whose dual is free abelian of rank $2^{\aleph_0}$. Does $\mathbb{Z}^\omega$ have a subgroup whose dual is free abelian of still larger rank (the largest possible being $2^{2^{\aleph_0}}$) ?

Q 10. [12, Question 5, p.97] Suppose α is an endomorphism of a group G, such that for every group H and homomorphism $f : G \to H$, there exists an endomorphism β_f of H such that $\beta_f f = f\alpha$. Is it then possible to choose such endomaps β_f for all f so as to constitute an endomorphism of the forgetful functor $(G \downarrow \mathbf{Group}) \to \mathbf{Group}$, i.e., so that for homomorphisms $G \xrightarrow{f} H_1 \xrightarrow{h} H_2$ we always have $\beta_{hf} h = h\beta_f$?

(By the results of [12, §1], this is equivalent to asking whether α must either be an inner automorphism of G or the trivial endomorphism.)

Q 11. [11, Question 16, p.157] Suppose G is a group, and $X,\ Y,\ Z$ are finite transitive G-sets, such that

$$\text{g.c.d.}(\text{card}(X),\ \text{card}(Y),\ \text{card}(Z)) \ = \ 1,$$

but such that none of the G-sets $X{\times}Y,\ Y{\times}Z,\ Z{\times}X$ is transitive (so that no two of $\text{card}(X)$, $\text{card}(Y),\ \text{card}(Z)$ are relatively prime).

What, if anything, can one conclude about the orbit-structure of the G-set $X \times Y \times Z$? (For the motivation, see [11], paragraph preceding this question, then the group-theoretic proof of [11, Theorem 1, p.139], and finally, the first paragraph of §2 on that page. But perhaps the question is, nonetheless, too vague to include in the *Kourovka Notebook*.)

Q 12. [9, group case of Question 4.4, p.362] Suppose B is a subgroup of the permutation group Σ on an infinite set Ω. Will the coproduct of two copies of Σ with amalgamation of B be embeddable in Σ?

In [10, §10] it is noted that the above coproduct need not be so embeddable by a map respecting B – for instance, if B is the stabilizer of a single element of Ω. But it is asked whether that, too, can be achieved under various assumptions on B; for instance, if B is the stabilizer of a subset of Ω of the same cardinality as Ω, or more generally, a subgroup of such a stabilizer.

Q 13. (i) [9, group case of Question 4.5, p.363] Do there exist a variety $\mathbf{V}$ of groups and a group $G \in \mathbf{V}$ such that the coproduct in $\mathbf{V}$ of two copies of G is embeddable in G, but the coproduct of three such copies is not?

(Given an embedding $G *_{\mathbf{V}} G \to G$, one might expect the induced map $G *_{\mathbf{V}} (G *_{\mathbf{V}} G) \to G *_{\mathbf{V}} G \to G$ to be an embedding. But this is not automatic, because in a general group variety $\mathbf{V}$, a map $G *_{\mathbf{V}} A \to G *_{\mathbf{V}} B$ induced by an embedding $A \to B$ is not necessarily again an embedding.)

(ii) [10, group case of Question 14(iii), p.863] If there exist $\mathbf{V}$ and G as in (i), does there in fact exist an example with $\mathbf{V}$ the variety of groups generated by G?

(iii) [10, group case of first paragraph of Question 13, p.861] Given a variety $\mathbf{V}$ of groups, let us define an equivalence relation on natural numbers by making m and n equivalent if the free groups in $\mathbf{V}$ on m and n generators each embed in the other. The equivalence classes will clearly be blocks of consecutive integers. Which block decompositions of the natural numbers can be so realized?

Q 14. (After [9, Question 9.1, p.386].) Let κ be an infinite cardinal. If a *residually finite* group G is embeddable in the full permutation group of a set of cardinality κ, must it be embeddable in the direct product of κ *finite* groups?

(The converse is clearly true: any such direct product is residually finite and embeddable in the indicated permutation group. Also, the statement asked for becomes true if one replaces "direct product of κ finite groups" by "direct product of 2^{2^κ} finite groups", since G has cardinality at most 2^κ, hence has at most 2^{2^κ} finite homomorphic images.)

Q 15. ([8, Question 9], restated in the language used by V. Tolstykh in [1, problem 16.88] to state [8, Question 8].) Let the *width* of a group (respectively, a monoid) H with respect to a generating set X mean the supremum over $h \in H$ of the least length of a group word (respectively, a monoid word) in elements of X expressing h. A group or monoid is said to have *finite width* if its width with respect to *every* generating set is finite. (A common finite bound for these widths is not required.)

Do there exist groups G having finite width as groups, but not as monoids?

Q 16. [7, Question 10, p.437] In [3] Baumslag and Shalen show that a group is infinite if it has a presentation in which all relations have the form $s^n = 1$, for various group words s, and a common exponent n which is a prime power, such that the number of generators in the presentation is at least 1 more than $1/n$ times the number of relations.

Can one get a more general criterion for a group to be infinite, again based on counting a relation of the form $s^n = 1$ as "$1/n$ of a relation" but with the "common exponent" assumption replaced by the condition that the exponents n so treated all be powers of a common prime p; or alternatively, with the common exponent requirement retained, but no requirement that it be a prime power; or, covering both these generalizations, merely the requirement that the set of exponents so treated be totally ordered under divisibility?

(For the need for some such restrictions, see [7, Introduction]. For another known result of the same sort, see [23], also summarized in [7, Introduction].)

Q 17. [6, Question 21, p.1548] Suppose $K < H < F$ are free groups of finite rank, such that $\mathrm{rank}(H) < \mathrm{rank}(K)$, but all proper subgroups of H which contain K have ranks $\geq \mathrm{rank}(K)$. Then is the inclusion of H in F the only homomorphism $H \to F$ fixing all elements of K ?

This question may be broken into two parts: (a) Is the inclusion of H in F the only *one-to-one* homomorphism $H \to F$ fixing all elements of K? (b) Is every homomorphism $H \to F$ fixing all elements of K one-to-one?

Q 18. If $\mathbf{X}$ is a class of groups, let $\mathbf{H}(\mathbf{X})$ denote the class of homomorphic images of groups in $\mathbf{X}$, let $\mathbf{S}(\mathbf{X})$ denote the class of groups isomorphic to subgroups of groups in $\mathbf{X}$, let $\mathbf{P}(\mathbf{X})$ denote the class of groups isomorphic to products of families of groups in $\mathbf{X}$, and let $\mathbf{P}_f(\mathbf{X})$ denote the class of groups isomorphic to products of *finite* families of groups in $\mathbf{X}$.

By the group case of Birkhoff's Theorem, $\mathbf{H}\,\mathbf{S}\,\mathbf{P}(\mathbf{X})$ is the variety of groups generated by $\mathbf{X}$.

(i) [5, Question 27, p.281] If $\mathbf{X}$ is a class of *metabelian* groups, must $\mathbf{H}\,\mathbf{S}\,\mathbf{P}_f(\mathbf{X}) \subseteq \mathbf{S}\,\mathbf{H}\,\mathbf{P}\,\mathbf{S}(\mathbf{X})$? (See preceding paragraph of [5] for motivation.)

The operators $\mathbf{H}$, $\mathbf{S}$, $\mathbf{P}$ make sense for more general algebraic structures, in particular, lattice-ordered groups, as in

(ii) (S. Comer, personal correspondence, cited in [5, Question 28(ii), p.281].) If $\mathbf{X}$ is a class of *lattice-ordered groups*, must $\mathbf{S}\,\mathbf{H}\,\mathbf{P}\,\mathbf{S}(\mathbf{X}) = \mathbf{H}\,\mathbf{S}\,\mathbf{P}(\mathbf{X})$?

Q 19. A *hyperidentity* for a class $\mathbf{X}$ of algebraic objects of a given type is a relation $\sigma = \tau$ in a family of operation-symbols $g_1, g_2, \dots$ of specified arities, which holds for every algebra $A \in \mathbf{X}$ and every assignment to each g_i of a *derived operation* of the corresponding arity [22].

[4, second sentence on p.65] If a variety $\mathbf{V}$ of groups satisfies a nontrivial monoid identity (a monoid identity other than those satisfied by the variety of all monoids), must $\mathbf{V}$ satisfy a hyperidentity not satisfied by the variety of all groups? (If one merely assumes that $\mathbf{V}$ satisfies a nontrivial group identity, the answer is no: by [4, Corollary 2] the variety of metabelian groups satisfies the same hyperidentities as the variety of all groups. But this is proved using the fact that that variety satisfies no nontrivial monoid identities.)

Q 20. [4, p.65, 2nd paragraph] If a monoid satisfies nontrivial identities, must its universal group satisfy the same identities? If not, must it at least satisfy some nontrivial group identities?

Remark on some questions not listed here. In [16], Isaacs and I studied the fixed rings of actions of groups on rings, and noted some open questions. Since these involve the structures of the rings more than those of the groups, they are probably not appropriate for the *Kourovka Notebook*.

As an easy-to-state example, here is a question of Herstein [18] that we looked at: If a ring R has an automorphism α of prime order p, such that all fixed elements of α lie in the center of R, must the commutator ideal of R be nil?

Herstein proves an affirmative answer for $p = 2, 3$. We prove in [16, Proposition 6.1] that with p generalized to an arbitrary positive integer n, the answer is affirmative if R has no n-torsion.

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In each reference below, the questions and/or sections where that work is cited are noted at the end, in parentheses.

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George M. Bergman, Department of Mathematics, University of California, Berkeley, CA 94720-3840, USA

E-mail address: gbergman@math.berkeley.edu

URL: https://math.berkeley.edu/~gbergman