1. Ross exercise 1.3.

2. Ross exercise 1.8.

3. The Fibonacci numbers are given by \( F(1) = 0, F(2) = 1, F(n + 2) = F(n + 1) + F(n) \) for \( n \in \mathbb{N} \). Define \( f(n) = \frac{1}{\sqrt{5}}(a^{n-1} - (1-a)^{n-1}) \) where \( a = \frac{\sqrt{5} + 1}{2} \). Let \( P_n \) be the proposition that “both \( F(n) = f(n) \) and \( F(n + 1) = f(n + 1) \)”. Use mathematical induction to show that \( P_n \) holds for all \( n \in \mathbb{N} \), thus deduce \( F(n) = f(n) \) for all \( n \).

4. Show that \( \sqrt{3} - \sqrt{2} \) is not rational.

5. Show \( ||a| - |b|| \leq |a-b| \) for all real numbers \( a, b \).


8. Let \( A, B \) be nonempty bounded sets in \( \mathbb{R} \) and define \( M = \{a \cdot b| a \in A, b \in B\} \). Is \( \text{sup} A \cdot \text{sup} B = \text{sup} M \)? Either prove or give a counterexample.
