1. Observe that the function \( \frac{z^2+1}{z} \) is analytic inside \( C \). Therefore 
\[
\int_C \frac{z^2+1}{z-2} \, dz = 2\pi i \left. \frac{z^2+1}{z} \right|_{z=2} = 5\pi i.
\]

2. Suppose \( f(z) \) is analytic at \( z = 0 \), then \( x^2 - iy^2 = f(z)e^{-z} \) is analytic at \( z = 0 \). By direct calculation, we find \( x^2 - iy^2 \) satisfies the Cauchy-Riemann equation only at \( x = -y \) which does not contain any neighborhood of \( z = 0 \). Thus \( f(z) \) is not analytic at \( z = 0 \).

3. Direct Calculation. \( g(x, y) = 3x^2y - y^3 \).

4. \( |f(z)| = |z^2 - 3z + 2| \leq |z|^2 + 3|z| + 2 \leq 6 \). Note that the equality could be achieved at \( z = -1 \).