Worksheet 5

1) a) $\frac{x^2+2x+5}{x-2} = \frac{x(x-2)+4(x-2)+13}{(x-2)} = x+4+\frac{13}{x-2}$ (can also do $u=x-2$)

$\int \frac{x^2+2x+5}{x-2}dx = \frac{x^2}{2}+4x+13\ln|x-2|+C$

b) $\frac{e^x}{e^x-e^{-x}} = \frac{e^x}{e^x}\frac{e^x}{e^x-e^{-x}} = \frac{e^{2x}}{e^{2x}-1}$

$\int \frac{e^x}{e^x-e^{-x}}dx = \int \frac{e^{2x}}{e^{2x}-1}dx$ $\quad u=e^{2x}-1$, $du=2e^{2x}dx$

$= \int \frac{1}{2u}du = \frac{1}{2}\ln|u|+C = \frac{1}{2}\ln|e^{2x}-1|+C$

c) $u=e^{e^x} \Rightarrow du = e^{e^x}e^x dx$ (can also do $u=e^x$ then $v=e^u$)

$\int e^{e^{e^x}}e^{e^x}e^x dx = \int e^u du = e^u + C = e^{e^{e^x}}+C$

2) a) $u=t^2+2t$, $du=(2t+2)dt$

$\int e^{t^2+2t}(t+1)dt = \int e^{t^2+2t}\cancel{(t+1)}\frac{du}{\cancel{2t+2}_{\,2}} = \int \frac{1}{2}e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{t^2+2t}+C$

$\int_{-2}^{-1} e^{t^2+2t}(t+1)dt = \left(\frac{1}{2}e^{1-2}\right) - \left(\frac{1}{2}e^{4-4}\right) = \frac{1}{2}e^{-1}-\frac{1}{2}$

b) $\int_1^4 \frac{x^2+3x}{\sqrt{x}}dx = \int_1^4 x^{3/2}+3x^{1/2}dx = \left(\frac{2}{5}x^{5/2}+2x^{3/2}\right)\Big|_1^4$

$= \left(\frac{2}{5}4^{5/2}+2\cdot4^{3/2}\right) - \left(\frac{2}{5}+2\right) = \frac{64}{5}+16-\frac{2}{5}-2 = 26+\frac{2}{5}$

3) a) $f(x) = \frac{x^3}{3}+\frac{1}{x}+C \Rightarrow \frac{4}{3}=f(1)=\frac{4}{3}+C \Rightarrow C=0 \Rightarrow f(x)=\frac{x^3}{3}+\frac{1}{x}$

b) $g(x) = \int 2^{x^3}x^2dx$ $\quad u=x^3$, $du=3x^2dx$

$= \int \frac{2^u}{3}du = \frac{2^u}{3\ln(2)}+C = \frac{2^{x^3}}{3\ln(2)}+C$

$\Rightarrow \frac{8}{\ln(2)} = g(1) = \frac{2}{3\ln(2)}+C$

$\Rightarrow C = \frac{22}{3\ln(2)}$

$g(x) = \frac{2^{x^3}}{3\ln(2)}+\frac{22}{3\ln(2)}$

c) $h(x)=\int \frac{x-2}{e^{4x-x^2}}dx$ $u=4x-x^2,\ du=(4-2x)dx$

$$= \int \frac{x-2}{e^{4x-x^2}}\frac{du}{4-2x} = \int \frac{-du}{2e^{4x-x^2}} = \int \frac{-du}{2e^u} = \int \frac{-e^{-u}}{2}du = \frac{e^{-u}}{2}+C$$

$$= \frac{e^{x^2-4x}}{2}+C \implies -5=h(0)=\frac{1}{2}+C \implies C=-11/2$$

$$h(x)=\frac{e^{x^2-4x}}{2}-\frac{11}{2}$$

4) a) $f(x)=0$ when $x=\frac{2\pm\sqrt{8}}{2}=1\pm\sqrt{2}$
then $f(x)>0$ in $[2, 1+\sqrt{2})$, $f(x)<0$ in $(1+\sqrt{2}, 3]$

Total Area $= \int_2^{1+\sqrt{2}} f(x)dx - \int_{1+\sqrt{2}}^3 f(x)dx$

$$= \left(-\frac{x^3}{3}+x^2+x\right)\Big|_2^{1+\sqrt{2}} - \left(-\frac{x^3}{3}+x^2+x\right)\Big|_{1+\sqrt{2}}^3$$

$$= 2\left(-\frac{(1+\sqrt{2})^3}{3}+(1+\sqrt{2})^2+1+\sqrt{2}\right)-\left(-\frac{8}{3}+6-\frac{27}{3}+12\right)$$

b) $g(x)>0$ for x in $[2,3]$
Total area $= \int_2^3 g(x)dx = \left(\frac{x^2}{2}+8x^{1/2}\right)\Big|_2^3$

$$= \left(\frac{9}{2}+8\sqrt{3}\right)-\left(2+8\sqrt{2}\right)=\frac{5}{2}+8(\sqrt{3}-\sqrt{2})$$