Worksheet #4

1) a) $f'(x) = \dfrac{\frac{x^2+1}{2x-3} - 2x\ln(2x-3)}{(x^2+1)^2} \Rightarrow f'(2) = \dfrac{5-4\ln(1)}{5^2} = 1/5$

Perpendicular Slope $= -5$. $(2, f(2)) = (2, 0)$

$y = -5x + 10$

b) $f'(x) = 3x^2 - 4x + 1 \Rightarrow f'(1) = 0$

Perpendicular slope is vertical. $(1, f(1)) = (1, 10)$

$x = 1$

c) $f'(x) = \ln(4)\, 4^{x^2+x}(2x+1) \Rightarrow f'(0) = \ln(4) . 5$

Perpendicular slope $= \dfrac{-1}{5\ln(4)}$. $(0, f(0)) = (0, 1)$

$y - 1 = \dfrac{-1}{5\ln(4)} x$

d) $f'(x) = \frac{1}{2} x^{-1/2} e^{3x} + 3x^{1/2} e^{3x} \Rightarrow f'(1) = \frac{e^3}{2} + 3e^3 = \frac{7}{2} e^3$

Perpendicular Slope $= -\dfrac{2}{7e^3}$. $(1, f(1)) = (1, e^3)$

$y - e^3 = -\dfrac{2}{7e^3}(x-1)$

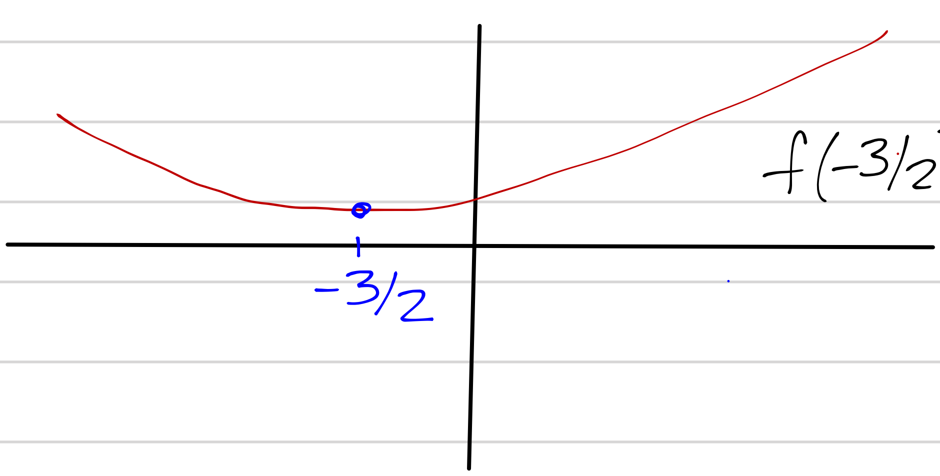
2) a) Domain: $(-\infty, +\infty)$

$f'(x) = (2x+3) . e^{x^2+3x+1}$. $f' = 0$ at $x = -3/2$

— $-3/2$ +

local min

$f''(x) = \left((2x+3)^2 + 2\right) e^{x^2+3x+1}$ $f'' > 0$ always concave up

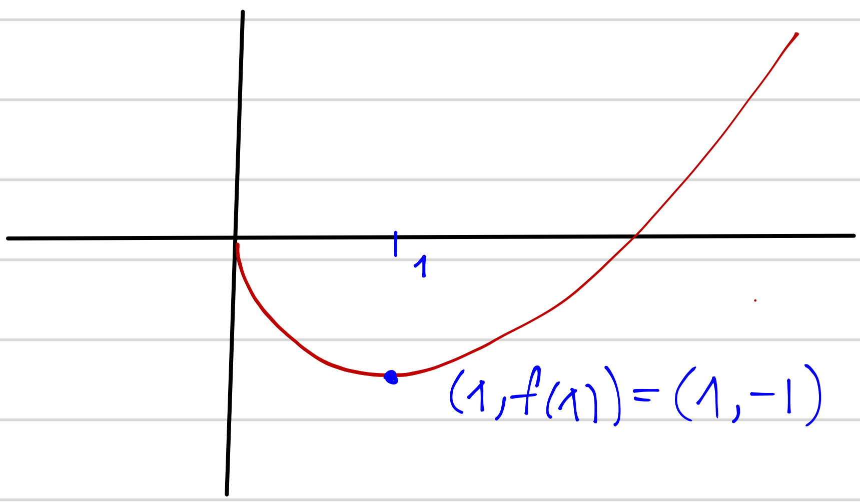


$f(-3/2) > 0$

b) Domain $(0,+\infty)$

$f'(x) = \ln(x)$ $\qquad$ $f'=0$ at $x=1$ $\qquad$ $\underset{1}{-\quad|\quad+}$ local min

$f''(x) = 1/x$ $\qquad$ $f'>0$ ($x>0$ because of Domain) Concave up



c) Domain $(-\infty,+\infty)$ $(e^{-t}\neq -1)$

$$f'(t)=\frac{e^{-t}}{(1+e^{-t})^2} \qquad f'>0$$

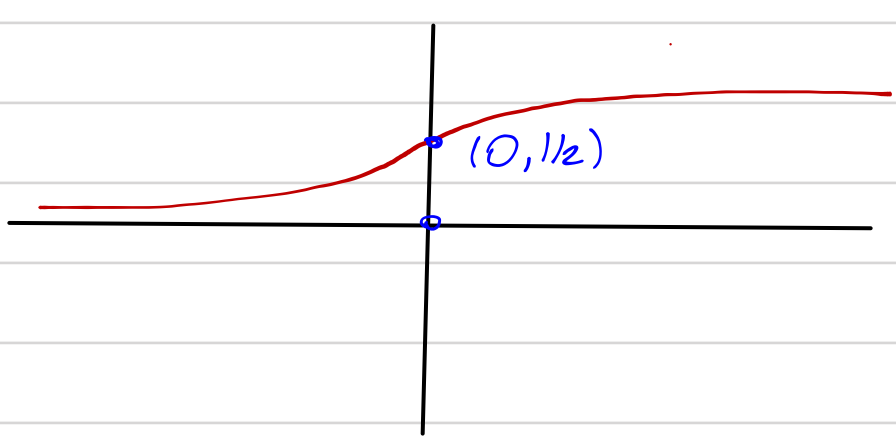
$$f''(t)=\frac{-e^{-t}(1+e^{-t})^2+2e^{-2t}(1+e^{-t})}{(1+e^{-t})^4}$$

$$=\frac{e^{-t}(1+e^{-t})(e^{-t}-1)}{(1+e^{-t})^4}=\frac{e^{-t}(e^{-t}-1)}{(1+e^{-t})^3}$$

$f''=0 \Rightarrow e^{-t}=1 \Rightarrow t=0$

$\underset{0}{+\quad|\quad-}$

c. up $\qquad$ c. down



d) Domain $(0,+\infty)$ (Because $x^{5/2}$)

$$f'(x)=\frac{\frac{5}{2}x^{3/2}(3x^2+2)-x^{5/2}(6x)}{(3x^2+2)^2} \qquad f'>0$$

$$=\frac{\frac{3}{2}x^{7/2}+\frac{5}{2}x^{3/2}}{(3x^2+2)^2}=\frac{x^{3/2}}{(3x^2+2)^2}\left(\frac{3}{2}x^2+\frac{5}{2}\right)$$

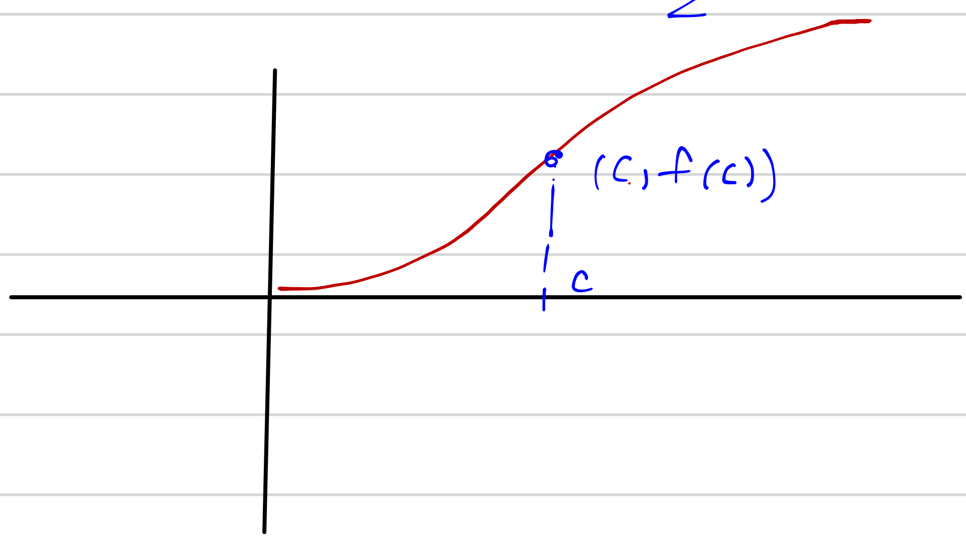
$$f''(x)=\frac{\left(\frac{21}{4}x^{5/2}+\frac{15}{4}x^{1/2}\right)(3x^2+2)^2-\left(3x^{7/2}+5x^{3/2}\right)(3x^2+2)(6x)}{(3x^2+2)^4}$$

$$=\frac{x^{1/2}}{(3x^2+2)^3}\left(\frac{63}{4}x^4+\frac{87}{4}x^2+\frac{15}{2}-18x^4-30x^2\right)$$

$$=\frac{x^{1/2}}{(3x^2+2)^3}\left(-\frac{9}{4}x^4-\frac{33}{4}x^2+\frac{15}{2}\right) \Rightarrow f''=0 \Rightarrow x^4+\frac{11}{9}x^2-\frac{10}{3}$$

$$x^2=\frac{-11/9\pm\sqrt{1201/81}}{2}$$

$$\Rightarrow x=\sqrt{\frac{\sqrt{1201/81}-11/9}{2}}=c \qquad \underset{+\quad c\quad -}{\overline{\quad\quad\quad}}$$



3) We need to make $f'(1)=0$

a) $f'(x)=kx^{k-1}+\frac{5}{2}x^{-1/2}+6x^{-3} \Rightarrow f'(1)=k+\frac{17}{2} \Rightarrow k=-17/2$

b) We need f to be defined at $x=1 \Rightarrow 1^3-3\cdot1^2+k\cdot x-1=0 \Rightarrow k=3$

$f(x)=\sqrt{(x-1)^3/(x-1)}=|x-1|$ which has 1 as a critical number

c) $f'(x)=\frac{1}{2\sqrt{\ln(x^2-kx+10)}}\cdot\frac{2x-k}{x^2-kx+10} \Rightarrow f'(1)=0$ if $k=2$ $(2x-k=0)$