Worksheet #3

1) a) $\frac{d}{dx}\left((x^2+3x+4)^{10}\right) = 9(x^2+3x+4)^{10} \cdot (2x+3)$ Chain rule

b) $\frac{d}{dx}\left((x^4+2x^2)^{-1/2}\right) = -\frac{1}{2}(x^4+2x^2)^{-3/2} \cdot (4x^3+4x)$ Chain rule

$$= \frac{-2(x^3+x)}{\sqrt{x^4+2x^2}^{\,3}} = \frac{-2x(x^2+1)}{|x|^3\sqrt{x^2+2}} \quad (\sqrt{x^2}=|x|)$$

c) $\frac{d}{dx}\left(2^{3^x}\right) = \left(\ln 2 . 2^{3^x}\right)\left(\ln 3 . 3^x\right)$ Chain rule

$\frac{d}{dx}(a^x) = \ln(a) . a^x$

$$= (\ln 2)(\ln 3)\, 2^{3^x} \cdot 3^x$$

d) $\frac{d}{dx}\left(\frac{e^x+e^{-x}}{e^x-e^{-x}}\right) = \frac{(e^x-e^{-x})(e^x-e^{-x}) - (e^x+e^{-x})(e^x+e^{-x})}{(e^x-e^{-x})^2}$ Quotient rule

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(e^{-x}) = -e^{-x}$

$$= \frac{(e^{2x}-2+e^{-2x})-(e^{2x}+2+e^{-2x})}{(e^x-e^{-x})^2} = \frac{-4}{(e^x-e^{-x})^2}$$

e) $\frac{d}{dx}\left(\frac{xe^x}{2x+1}\right) = \frac{(xe^x)'(2x+1)-(xe^x).(2)}{(2x+1)^2}$ Quotient rule

Product rule

$$= \frac{(e^x+xe^x)(2x+1)-(xe^x).2}{(2x+1)^2}$$

$$= \frac{e^x(1+x(2x+1)-2x)}{(2x+1)^2} = \frac{e^x(2x^2-x+1)}{(2x+1)^2}$$

f) $\frac{d}{dx}(x^x) = \frac{d}{dx}\left(e^{x\ln(x)}\right) = e^{x\ln(x)} \cdot (x\ln(x))'$ Chain rule $\left(\frac{d}{dx}(e^x) = e^x\right)$

Hint!

$= e^{x\ln x}\left(\ln(x) + x \cdot \frac{1}{x}\right)$ Product rule $\left(\frac{d}{dx}\ln x = \frac{1}{x}\right)$

$= \underline{e^{x\ln x}(\ln(x) + 1)}$

g) $\frac{d}{dx}\left(x^2\ln(x) - x\ln^2(x)\right) = \left[2x \cdot \ln(x) + x^2 \cdot \frac{1}{x}\right] - \left[\ln^2(x) + x(\ln^2(x))'\right]$

Two Product rules

$= \left[2x\ln(x) + x\right] - \left[\ln^2(x) + x\left(\underbrace{2\ln(x) \cdot \frac{1}{x}}\right)\right]$ Chain rule on $(\ln^2(x))'$

$\left(\frac{d}{dx}(\ln(x)) = \frac{1}{x}\right)$

$= \underline{2x\ln(x) + x - \ln^2(x) - 2\ln(x)}$

2) $(f \circ g)'(3) = f'(g(3)) \cdot g'(3) = f'(2) \cdot g'(3) = 4 \cdot 5 = \underline{20}$

$g(3) = 2$

3)

a) $\frac{d}{dx}\left(\frac{3x-4}{e^{2x}+1}\right) = \frac{3(e^{2x}+1) - (3x-4)(2e^{2x})}{(e^{2x}+1)^2}$ Quotient rule $\left(\frac{d}{dx}(e^{2x}) = 2e^{2x}\right)$

$$= \frac{3 + e^{2x}(7-3x)}{(e^{2x}+1)^2}$$

Slope at $x=0$ is the derivative when we replace $x=0$

$$= \frac{3 + e^{2(0)}(7-3(0))}{(e^{2(0)}+1)^2} = \frac{3+4}{2^2} = \underline{\frac{7}{4}}$$

b) $\frac{d}{dx}\left((5x^2-4)^9 \cdot (x^2+3x-4)^8\right) = \left[(5x^2-4)^9\right]'(x^2+3x-4)^8 + (5x^2-4)^9\left[(x^2+3x-4)^8\right]'$

Product rule

$= 9(5x^2-4)^8(10x)(x^2+3x-4)^8 + (5x^2-4)^9 \cdot 8(x^2+3x-4)^7(2x+3)$ Two chain rules

$= (5x^2-4)^8(x^2+3x-4)^7\left[90x(x^2+3x-4) + 8(5x^2-4)(2x+3)\right]$ Factoring

Slope at $x=1$: (let's replace now that looks simple enough)

$$= 1^8 \cdot 0^7 \left[90(1)(0) + 8(1)\cdot(0)\right] = \underline{0}$$

we have a 0 multiplying

c) $$\frac{d}{dx}\left(\ln\left(x^{3/2}(x^2+1)^{1/2}\right)\right) = \frac{1}{x^{3/2}(x^2+1)^{1/2}} \cdot \left(x^{3/2}(x^2+1)^{1/2}\right)'$$ Chain rule

$$= \frac{1}{x^{3/2}(x^2+1)^{1/2}}\left(\frac{3}{2}x^{1/2}(x^2+1)^{1/2} + x^{3/2} \cdot \underbrace{\frac{1}{2}\cdot(x^2+1)^{-1/2}\cdot(2x)}_{\text{chain rule of } ((x^2+1)^{1/2})'}\right)$$ Product rule

Only because we want the slope at $x=4$, let's replace and then simplify

$$\begin{pmatrix}\text{Slope at}\\ x=4\end{pmatrix} = \frac{1}{8(17)^{1/2}}\left(\frac{3}{\cancel{2}}\cdot\cancel{2}\cdot(17)^{1/2} + \overset{4}{\cancel{8}}\cdot\frac{1}{\cancel{2}}\,(17)^{-1/2}\cdot 8\right)$$

$$= \frac{3}{8} + \frac{4}{17} = \underline{\frac{83}{136}}$$

$\left(17^{1/2}\cdot 17^{1/2} = 17\right)$