Solution Worksheet #2

1) a) $= \lim_{x\to 3} \frac{(x-2)\cancel{(x-3)}}{(x+1)\cancel{(x-3)}} = \lim_{x\to 3} \frac{x-2}{x+1} = \frac{3-2}{3+1} = \frac{1}{4}$

b) $= \lim_{x\to -1} \frac{x-2}{x+1} = \frac{-3}{0}$ infinity

$\lim_{x\to -1^+} \frac{x-2 \longrightarrow -3<0}{x+1 \longrightarrow x+1>0\ (x>-1)} = -\infty$

$\lim_{x\to -1^-} \frac{x-2 \longrightarrow -3<0}{x+1 \longrightarrow x+1<0\ (x<-1)} = +\infty$

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c) $= \ln(3(2)-1) = \ln(5)$

d) $\lim_{x\to +\infty} \frac{1x^2-x-5 \longrightarrow \deg = 2}{2x^2+4 \longrightarrow \deg = 2} = \frac{1}{2}$

or $\lim_{x\to +\infty} \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \frac{x^2-x-5}{2x^2+4} = \lim_{x\to +\infty} \frac{1-\frac{1}{x}-\frac{5}{x^2}}{2+\frac{4}{x^2}} = \frac{1-0-0}{2+0} = \frac{1}{2}$

then $\lim_{x\to +\infty} e^{\left(\frac{x^2-x-5}{2x^2+4}\right)} = e^{\left(\lim_{x\to +\infty} \frac{x^2-x-5}{2x^2+4}\right)} = e^{\frac{1}{2}}$

2) a) $\lim_{x\to 1^+} \lfloor x \rfloor = 1$ since $\lfloor x \rfloor = 1$ for $1<x<2$

b) $\lim_{x\to 1^-} \lfloor x \rfloor = 0$ since $\lfloor x \rfloor = 0$ for $0<x<1$

c) $\lim_{x\to \frac{1}{2}} \lfloor x \rfloor = 0$ since $\lfloor x \rfloor = 0$ for $0<x<1$

3) For example, $\pi \approx 3.14$ then $\pi^2 + 1 \approx (3.14)^2 + 1 = 10.8596$

4) a) $\lim_{x\to +\infty} \frac{3x+4 \longrightarrow \deg = 1}{\sqrt{x^2-2x+3} \longrightarrow \deg = \frac{2}{2} = 1\ (\sqrt{x^2})} = \frac{3}{\sqrt{1}} = 3$

or $= \lim_{x\to +\infty} \frac{\frac{1}{x}}{\frac{1}{x}} \frac{3x+4}{\sqrt{x^2-2x+3}} = \lim_{x\to +\infty}{}^{*} \frac{3+\frac{4}{x}}{\sqrt{1-\frac{2}{x}+\frac{3}{x^2}}} = \frac{3+0}{\sqrt{1-0+0}} = 3$

b) $$= \lim_{x\to+\infty} \frac{1}{\sqrt{x+1}-\sqrt{x}} \cdot \frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}} = \lim_{x\to+\infty} \frac{\sqrt{x+1}+\sqrt{x}}{(x+1)-x}$$

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$$= \lim_{x\to+\infty} \sqrt{x+1}+\sqrt{x} = +\infty$$

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c) $$\lim_{x\to+\infty} \sqrt{x^2+4x+1}-x \cdot \frac{\sqrt{x^2+4x+1}+x}{\sqrt{x^2+4x+1}+x} = \lim_{x\to+\infty} \frac{x^2+4x+1-x^2}{\sqrt{x^2+4x+1}+x}$$

$$= \lim_{x\to+\infty} \frac{4x+1}{\sqrt{1x^2+4x+1}+1x} \begin{array}{l} \longrightarrow \text{deg } 1 \\ \longrightarrow \text{deg } 1 \end{array} = \frac{4}{\sqrt{1}+1} = \frac{4}{2} = 2$$

or $$= \lim_{x\to+\infty} \frac{\frac{1}{x}}{\frac{1}{x}} \frac{(4x+1)}{(\sqrt{x^2+4x+1}+x)} = \lim_{x\to+\infty} \frac{4+\frac{1}{x}}{\sqrt{1+\frac{4}{x}+\frac{1}{x^2}}+1} = \frac{4+0}{\sqrt{1}+1}$$

$*$

$$= 2$$

Some used properties: ☆ $(a-b)(a+b) = a^2-b^2$

□ $(\sqrt{a})^2 = a$

$*$ $\frac{1}{a}\sqrt{b} = \sqrt{\frac{b}{a^2}}$