Worksheet #1

1) V: vertex of P $\quad y=f(x)=3x^2-12x+5$

$V=\left(-\left(\frac{-12}{2\cdot 3}\right), f\left(-\left(\frac{-12}{2\cdot 3}\right)\right)\right)=(2, f(2))=(2,-7)$ vertex formula

$\Rightarrow L: \; y-(-7)=3(x-2)$ line formula

$L: \; y=3x-13$

$L\cap P: \; y=3x-13=3x^2-12x+5$

$\Rightarrow \; 0=3x^2-15x+18=(3x-9)(x-2)$

So $x=2$ or $x=3$

$x=2$ is the vertex $V=(2,-7)$

$x=3$ is $(3, 3(3)-13)=(3,-4)$

2) $C(x)$: cost of "x" garments $\Rightarrow C(x)=50+10x$

If p is the selling price, then $R(x)$ (revenue of x garments) is

$R(x)=px$

Breaking even for 20 garments: $R(20)=C(20)$

$\Rightarrow \; 20p=50+10(20)=250$

$\Rightarrow \; p=12{,}5$

Each extra garment costs \$10 and sells at \$12,5, so the profit is \$2,5.

3) a) Look out for $x^2-5x+4=0$ (no quotient by 0)

$0=x^2-5x+4=(x-1)(x-4) \Rightarrow x=1$ or $x=4$

Domain $(-\infty,1)\cup(1,4)\cup(4,+\infty)$

b) Look out for $x^2+x+2<0$ (no $\sqrt{}$ of a negative number)

$x^2+x+2=0$ has no solutions, this is because the solutions are $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ and in this case $b^2-4ac=-7$. quadratic formula

Since this parabola looks up, has to be positive.

Alternatively $x^2+x+2 = x^2+x+\frac{1}{4}+\left(2-\frac{1}{4}\right) = \left(x+\frac{1}{2}\right)^2+\frac{7}{4} > 0$

Domain: $\mathbb{R}$ (x^2+x+2 is always positive

c) Look out for $x^2-5x+4=0$ and $\frac{x^2+x+2}{x^2-5x+4} < 0$

From (a) $x^2-5x+4=0$ corresponds to $x=1$ or $x=4$ $(\square)$

From (b) $x^2+x+2>0$, so $\frac{x^2+x+2}{x^2-5x+4}$ is negative if and only if

x^2-5x+4 is negative. Now $x^2-5x+4=(x-1)(x-4)<0$

if $x \leq 1 \Rightarrow (x-1) \leq 0,\ x-4 \leq 0 \Rightarrow (x-1)(x-4) \geq 0$

if $1<x<4 \Rightarrow (x-1)>0,\ x-4<0 \Rightarrow (x-1)(x-4)<0$ $(\star)$

if $4 \leq x \Rightarrow (x-1) \geq 0,\ (x-4) \geq 0 \Rightarrow (x-1)(x-4) \geq 0$

Then from $(\square)$, $(\star)$, $x=1$, $x=4$, $1<x<4$ can't be on the domain

Domain $(-\infty,1) \cup (4,+\infty)$

4)

$9=3^2$ $(a^b)^c=a^{bc}$

a) $3^{x^2+1} = 9^{2x} = (3^2)^{2x} = 3^{4x}$

Now since $3^{x^2+1} = 3^{4x} \Rightarrow x^2+1=4x$ ($a^x=a^y \Rightarrow x=y$ if $a \neq 1$)

$\Rightarrow x^2-4x+1=0$

Solutions $x = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

$x = 2+\sqrt{3}$ or $x = 2-\sqrt{3}$

$4=2^2$ $(a^b)^c=a^{bc}$

b) $3^{2x} = 4^x = (2^2)^x = 2^{2x}$ $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$

Now $3^{2x} = 2^{2x} \Rightarrow \frac{3^{2x}}{2^{2x}} = 1 \Rightarrow \left(\frac{3}{2}\right)^{2x} = 1$

then since $\left(\frac{3}{2}\right)^{2x}=1 \Rightarrow 2x=0 \Rightarrow x=0$

$a+a=2a \qquad a^x \cdot a^y = a^{x+y}$

c) $2^x + 2^x = 2 \cdot 2^x = 2^{x+1}$

$\left(\frac{1}{16}\right)^x = \left(\frac{1}{2^4}\right)^x = (2^{-4})^x = 2^{-4x}$ $\quad > \; 2^{x+1} = 2^{-4x}$

$16 = 2^4 \qquad \frac{1}{x} = x^{-1}$

Since $2^{x+1} = 2^{-4x} \Rightarrow x+1 = -4x \Rightarrow 5x = -1 \Rightarrow x = -1/5$

$a^x = a^y \Rightarrow x = y$ for $a \neq 1$

5)

$4 = 2^2 \qquad (a^b)^c = a^{bc} \qquad \log_a(x^b) = b\log_a x$

a) $\log_5(4^2) = \log_5((2^2)^2) = \log_5(2^4) = 4\log_5 2$

$(\log_a b)(\log_b c) = \log_a c$

b) $(\log_7 11)(\log_{11} 49) = \log_7 49 = 2$ $\quad (\log_a x = y \Leftrightarrow a^y = x)$

$b\log_a x = \log_a(x^b) \qquad 3^2 = 9 \qquad (\log_a b)(\log_b c) = \log_a c$

c) $2(\log_2 3)(\log_9 16) = (\log_2 3^2)(\log_9 16) = (\log_2 9)(\log_9 16) = \log_2 16$

$= 4$ $\quad (\log_a x = y \Leftrightarrow a^y = x)$

d) $e^{x(\ln x)} = (e^{\ln x})^x = x^x$

$a^{bc} = (a^b)^c \qquad e^{\ln a} = a$