Quiz # 3

10/10

Name:

Section:

1) Derivate the following functions (6 points each)

a) $f(x) = \dfrac{\sqrt{x}}{x^2+1}$

b) $g(x) = \left(5x^4 - \sqrt[3]{x}\right)^2$

By quotient rule

$$f'(x) = \frac{\frac{1}{2}x^{-1/2}(x^2+1) - x^{1/2}(2x)}{(x^2+1)^2}$$

$$f'(x) = \frac{-\frac{3}{2}x^{3/2} + \frac{1}{2}x^{-1/2}}{(x^2+1)^2}$$

By Chain rule

$$g'(x) = 2(5x^4 - x^{1/3})(20x^3 - \tfrac{1}{3}x^{-2/3})$$

2) Let $f(x) = x^6$, $g(x) = x + K$, where K is an unknown constant.

a) Derivate $f(g(x))$ (5 points)

b) Find K so that the tangent line at $x=7$ is horizontal. (3 points)

a) By Chain rule

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x) = 6(x+K)^5$$

b) Horizontal tangent line $\longleftrightarrow$ $\frac{d}{dx}(f(g(x)))$ is 0 at $x=7$

$$\Rightarrow 6(7+K)^5 = 0 \Rightarrow K = -7$$

Quiz #5 11/16

Name:

Section:

1) Given the following implicit equation: $q^6 - 3q + e^p = e^2 - 2$

a) Find $\frac{dq}{dp}$ using implicit differentiation (6 points)

b) If near $q=1$, p and q model price and demand, would you recommend to increase or decrease price to increase revenue? (4 points)

(Hints: Calculate Elasticity. Also $e > 2$)

a) $\frac{dq}{dp}(6q^5 - 3) + e^p = 0 \Rightarrow \frac{dq}{dp} = \frac{-e^p}{6q^5 - 3}$

b) $q = 1 \Rightarrow 1 - 3 + e^p = e^2 - 2 \Rightarrow e^p = e^2 \Rightarrow p = 2$

$E = \frac{-p}{q}\frac{dq}{dp} = -\frac{2}{1}\left(\frac{-e^2}{3}\right) = \frac{2e^2}{3} > \frac{8}{3} > 1$. Decrease price

2) a) Verify that $f(x) = x\ln x - x$ is an antiderivative of $g(x) = \ln(x)$

Then find ALL antiderivatives of $g(x)$ (5 points)

b) Find $\int \frac{x^2 - \sqrt{x}}{x}\,dx$ (5 points)

a) $f'(x) = \ln(x) + x \cdot \frac{1}{x} - 1 = \ln(x) = g(x)$

then $\int g(x)\,dx = x\ln x - x + C$

b) $= \int x - x^{-1/2}\,dx = \frac{x^2}{2} - \frac{x^{1/2}}{1/2} + C = \frac{x^2}{2} - 2x^{1/2} + C$

Quiz #6 11/28

Name:

Section:

1) Solve the definite integrals (5 points each)

a) $\int_0^1 x^2 \cdot \sqrt{x+1}\, dx$

$u = x+1$, $du = dx$

$\int (u-1)^2 u^{1/2} du$

$= \int u^{5/2} - 2u^{3/2} + u^{1/2} du$

$= \frac{2}{7} u^{7/2} - \frac{4}{5} u^{5/2} + \frac{2}{3} u^{3/2} + C$

$= \frac{2}{7}(x+1)^{7/2} - \frac{4}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2} + C$

$\Rightarrow \int_0^1 x^2 \sqrt{1+x}\, dx = \left[\frac{2}{7} 2^{7/2} - \frac{4}{5} 2^{5/2} + \frac{2}{3} 2^{3/2}\right] - \left[\frac{2}{7} - \frac{4}{5} + \frac{2}{3}\right]$

b) $\int_{\ln(2)}^{\ln(4)} \frac{e^x}{e^x+3}\, dx$

$u = e^x + 3$

$du = e^x dx$

$\int \frac{du}{u} = \ln|u| + C$

$= \ln|e^x+3| + C$

$\int_{\ln(2)}^{\ln(4)} \frac{e^x}{e^x+3} dx = \left(\ln|e^x+3|\right)\Big|_{\ln(2)}^{\ln(4)}$

$= \ln(7) - \ln(5)$

$= \ln(7/5)$

2) a) Find $f(x)$ such that $f'(x) = \frac{3x-1}{\sqrt[3]{x}}$, $f(1) = 2$ (5 points)

b) Calculate the Net Area under the curve $y = f(x)$ from $x=0$ to $x=1$. (5 points)

a) $f'(x) = 3x^{2/3} - x^{-1/3} \Rightarrow f(x) = \frac{9}{5} x^{5/3} - \frac{3}{2} x^{2/3} + C$

$2 = f(1) = \frac{9}{5} - \frac{3}{2} + C \Rightarrow 2 = \frac{3}{10} + C \Rightarrow C = 17/10$

b) Net Area $= \int_0^1 f(x) dx = \int_0^1 \left(\frac{9}{5} x^{5/3} - \frac{3}{2} x^{2/3} + \frac{17}{10}\right) dx$

$= \left(\frac{27}{40} x^{8/3} - \frac{9}{10} x^{5/3} + \frac{17}{10} x\right)\Big|_0^1$

$= \left(\frac{27}{40} - \frac{9}{10} + \frac{17}{10}\right) - (0) = 59/40$