QUALIFYING EXAM SYLLABUS

Franco Vargas Pallete
franco@math.berkeley.edu

Committee: Ian Agol, John W. Lott (Chair), Maciej Zworski, Ronald Gronsky (Department of Materials Science and Engineering).

Algebraic Topology

- Fundamental group, Van Kampen's theorem, Covering Spaces.
- Homology: Singular and cellular homology, excision, long exact sequence of a pair, Mayer-Vietoris sequence. Universal Coefficient theorem. Brouwer fixed point theorem, hairy ball theorem, Borsuk-Ulam theorem.
- Cohomology:Singular and cellular cohomology, cup/cap products. Universal Coefficient theorem, Kunneth formula, Poincare duality, Thom isomorphism.
- Homotopy Theory: Higher homotopy groups, long exact sequence of a triple, Whitehead's theorem. Cellular and CW approximations. Eilenberg-MacLane spaces. Hurewicz theorem. Fiber bundles and fibrations.
- Characteristic Classes: Stiefel-Whitney, Euler, Chern.

References: A. Hatcher, Algebraic Topology (Chapters 1, 2, 3.1-3.3, 4.1-4.3) Milnor and Stasheff, Characteristic Classes

Riemannian Geometry

- Differentiable manifolds, Riemannian Metrics.
- Affine and Riemannian connections.
- Geodesics: Geodesic flow, minimizing properties of geodesics, convex neighborhoods.
- Curvature: Tensor curvature, sectional and Ricci curvature.
- Jacobi fields: Jacobi equation, conjugate points, second variation formula.
- Second fundamental form and the fundamental equations.
- Complete manifolds: Hopf-Rinow and Hadamard theorems.

- Spaces of constant curvature.
- Curvature and Topology: Rauch comparison theorem. Classical results: Bonnet-Myers, Synge-Weinstein, Preissman.

References: M. Do Carmo Riemannian Geometry
P. Petersen Riemannian Geometry
J. Lee Riemannian Manifolds: An Introduction to Curvature

Partial Differential Equations (Minor)

- Four important Linear PDE: Transport, Laplace's, Heat and Wave equations .
- Non-Linear First Order PDE: Method of Characteristics, Euler- Lagrange Equation, Hopf-Lax formula. Introduction to Hamilton-Jacobi equations.
- Theory of Distributions: Distribution, properties. Convolution, Fourier transform.
- Sobolev Spaces: Definitions, Approximation, Extensions, Traces. Sobolev inequalities, Compactness, Poincare's inequality.
- Second-order Elliptic Equations: Weak solutions, Existence, Regularity. Maximum principles, Eigenvalues and eigenfunctions.
- Second-order Parabolic and Hyperbolic equations: Definitions, Existence, Regularity, Maximum principle.
- Calculus of Variations: First and second variation, existence of minimizers. Regularity and constraints.
- Hamilton-Jacobi Equations: Viscosity solutions, uniqueness.

References: L. C. Evans Partial Differential Equations (Chapters 2,3.1-3.3, 5, 6, 7.1, 7.2, 8.1-8.5, 10.1, 10.2)
L. Hormander The Analysis of Linear Partial differential Operators I