MATH 53 DISCUSSION SECTION PROBLEMS – 4/4/23

Review for midterm 2:

1. Auroux practice exam A

1. Let $(\overline{x}, \overline{y})$ be the center of mass of the triangle with vertices $(-2, 0)$, $(0, 1)$, $(2, 0)$ and uniform density $\rho = 1$. Write an integral formula for $\overline{y}$. Do not evaluate the integral(s), but write explicitly the integrand and limits of integration.
2. Find the polar moment of inertia I_0 of the unit disk with density equal to the distance from the y-axis.
5. Consider the region R in the first quadrant bounded by the curves $y = x^2$, $y = x^2/5$, $xy = 2$, and $xy = 4$.
   (a) Compute $dxdy$ in terms of $dudv$ if $u = x^2/y$ and $v = xy$.
   (b) Express the area of R as a double integral in uv coordinates and evaluate it.
8. Let C be the portion of the cylinder $x^2 + y^2 \leq 1$ lying in the first octant $(x, y, z \geq 0)$ and below the plane $z = 1$. Set up a triple integral in *cylindrical coordinates* which gives the moment of inertia of C about the z-axis; assume the density to be $\rho = 1$. (Give integrand and limits of integration, but do not evaluate.)
9. A solid sphere S of radius a is placed above the xy-plane so that it is tangent at the origin and its diameter lies along the z-axis. Set up a triple integral in *spherical coordinates* which gives the volume of the portion of the sphere S lying above the plane $z = a$. (Give integrand and limits of integration, but do not evaluate.)

2. Auroux practice exam B

1. (a) Draw a picture of the region of integration of $\int_0^1 \int_x^{2x} dydx$.
   (b) Exchange the order of integration to express the integral of part (a) in terms of integration in the order $dxdy$.
2. (a) Find the mass M of the half-annulus $1 \leq x^2 + y^2 \leq 9$, $y \geq 0$, with density $\rho = \frac{y}{x^2+y^2}$.
   (b) Express the x-coordinate of the center of mass, $\overline{x}$, as an iterated integral. (Write explicitly the integrand and limits of integration.) Without evaluating the integral, explain why $\overline{x} = 0$.
5. Find the volume of the region enclosed by the plane $z = 4$ and the surface $z = (2x-y)^2 + (x+y-1)^2$. (Suggestion: change of variables.)
8. (a) Show that S lies in the upper half space $(z \geq 0)$.
   (b) Write out the equation for the surface in spherical coordinates.
   (c) Using the equation obtained in (b), give an iterated integral, with explicit integrand and limits of integration, which gives the volume of the region inside this surface. Do not evaluate the integral.

3. Agol practice exam

1. Find the volume of the solid that lies under the hyperbolic paraboloid $z = 3y^2 - x^2 + 2$ and above the rectangle $R = [-1, 1] \times [1, 2]$ in the xy-plane.
2. Evaluate the integral by reversing the order of integration:
$$\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2) dxdy.$$
3. Let R be the region $R = \{(x, y) : 1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq x\}$. Evaluate the integral by converting to polar coordinates:
$$\int\int_R \arctan(y/x) dA.$$

4. Find the volume and centroid of the solid E that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$, using cylindrical or spherical coordinates, whichever seems more appropriate.
5. Let R be the parallelogram with vertices $(-1, 3)$, $(1, -3)$, $(3, -1)$, and $(1, 5)$. Use the transformation $x = (u + v)/4$, $y = (v - 3u)/4$ to evaluate the integral
$$\int\int_R (4x + 8y) dA.$$