MATH 53 DISCUSSION SECTION PROBLEMS – 4/20/23

1. Parametric surfaces and their areas

(1) **(textbook 16.6.5)** Identify the surface with the vector equation $\mathbf{r}(s,t) = \langle s\cos t, s\sin t, s\rangle$.
(2) **(textbook 16.6.19)** Find a parametric representation for the plane through the origin that contains the vectors $\mathbf{i} - \mathbf{j}$ and $\mathbf{j} - \mathbf{k}$.
(3) **(textbook 16.6.25)** Find a parametric representation for the part of the sphere $x^2 + y^2 + z^2 = 36$ that lies between the planes $z = 0$ and $z = 3\sqrt{3}$.
(4) **(textbook 16.6.37)** Find an equation of the tangent plane to $\mathbf{r}(u,v) = u^2\mathbf{i} + 2u\sin v\mathbf{j} + u\cos v\mathbf{k}$ at the point corresponding to $u = 1$, $v = 0$.
(5) **(textbook 16.6.48)** Find the area of the helicoid with vector equation $\mathbf{r}(u,v) = u\cos v\mathbf{i} + u\sin v\mathbf{j} + v\mathbf{k}$, $0 \le u \le 1$, $0 \le v \le \pi$.

2. Surface Integrals of functions

(6) True/false practice:
  (a) The angular coordinates (ϕ, θ) used in spherical polar coordinates are often a good choice for parametrizing surfaces which have some rotational symmetries.
  (b) As with our integral formula for surface area, we have a shortcut formula for surface integrals of functions $g(x,y,z)$ over graphs $z = f(x,y)$.
(7) **(textbook 16.7.17 with a typo, oops)** Evaluate $\iint_S (x^2y + y^2z)\,dS$, where S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \ge 0$.
(8) **(a cross between textbook 16.7.9 and 16.7.10, oops)** Find $\iint_S x^2yz\,dS$, where S is the part of the plane $2x + 2y + z = 4$ that lies in the first octant.

3. Notes

Original author: James Rowan.

All problems labeled "textbook" come from Stewart, James, *Multivariable Calculus: Math 53 at UC Berkeley*, 8th Edition, Cengage Learning, 2016.

Problems marked **(*)** are challenge problems, with problems marked **(**)** especially challenging problems.