

MATH 53 DISCUSSION SECTION PROBLEMS – 3/9/23

1. DOUBLE INTEGRALS OVER GENERAL REGIONS

(1) True/false practice:

(a) The iterated integral

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1-x^2-y^2} dy dx$$

will give us the volume of the top hemisphere of a sphere of radius 1.

(b) If we need to find a double integral over a region that is of neither type I nor type II, there is still hope.

(2) Describe the following regions in the plane as i) a type I region and ii) a type II region:

(a) The region above the parabola $y = x^2$ and below the line $y = 1$.

(b) The region between the line $y = 2x$ and the x -axis for x between 0 and 1.

(c) The region inside the circle with radius 1 and center $(1, 2)$.

(3) **(textbook 15.2.3)** Evaluate the iterated integral

$$\int_0^1 \int_0^y x e^{y^3} dx dy$$

and sketch the domain of integration. What would happen if you tried to do the integral in the other order?

(4) **(textbook 15.2.7)** Evaluate the double integral

$$\iint_D \frac{y}{x^2 + 1} dA, \quad D = \{(x, y) | 0 \leq x \leq 4, 0 \leq y \leq \sqrt{x}\}.$$

(5) **(textbook 15.2.23)** Find the volume of the solid lying under the plane $3x + 2y - z = 0$ and above the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

(6) **(textbook 15.2.59)** Give upper and lower bounds for the value of the integral

$$\iint_S \sqrt{4 - x^2 y^2} dA, \quad S = \{(x, y) | x^2 + y^2 \leq 1, x \geq 0\}.$$

(7) **(from an old exam)** Evaluate the double integral

$$\int_0^1 \int_{x^2}^1 e^{y^{3/2}} dy dx.$$

Note (since the typesetting might not be clear) that the exponent is the function $y^{3/2}$.

2. NOTES

Original author: James Rowan.

All problems labeled “textbook” come from Stewart, James, *Multivariable Calculus: Math 53 at UC Berkeley*, 8th Edition, Cengage Learning, 2016.

Problems marked (*) are challenge problems, with problems marked (**) especially challenging problems.