MATH 53 DISCUSSION SECTION PROBLEMS – 3/7/23

1. Lagrange multipliers

(1) True/false practice:
  (a) When using Lagrange multipliers to find the maximum of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$, we always get a system of linear equations in x, y, z, λ which we will immediately know how to solve.
  (b) The geometric intuition behind the method of Lagrange multipliers is that the maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$, if they exist, should correspond to the points where the level surfaces of f are tangent to the constraint surface $g(x, y, z) = k$.

(2) **(textbook 14.8.5)** Given that the extreme value problem has a solution with both a maximum value and a minimum value, use Lagrange multipliers to find the extreme values of $f(x, y) = xy$ subject to the constraint $4x^2 + y^2 = 8$.

(3) **(textbook 14.8.21)** Find the extreme values of $f(x, y) = x^2+y^2+4x-4y$ on the region $x^2+y^2 \leq 9$.

(4) **((*), textbook 14.8.49)** Find the maximum value of $f(x_1, x_2, \ldots, x_n) = \sqrt[n]{x_1 x_2 \cdots x_n}$ given that $x_1, x_2, \ldots, x_n$ are positive numbers and $x_1 + x_2 + \cdots + x_n = n$. Why does this imply that for all positive numbers $x_1, x_2, \ldots, x_n$,

$$\sqrt[n]{x_1 x_2 \cdots x_n} \leq \frac{x_1 + x_2 + \cdots + x_n}{n}?$$

When does equality hold?

2. Double integrals over rectangles

(5) True/false practice:
  (a) Analogous to the midpoint rule for approximating integrals of functions of one variable, we have a midpoint rule for approximating double integrals.
  (b) When expressing a double integral of a continuous function $f(x, y)$ over a rectangular region R of the form $\{(x, y)|a \leq x \leq b, c \leq y \leq d\}$ as an iterated integral, we do get to make at least one choice that could make our lives easier.
  (c) $\int_0^1 \int_0^x e^{xy} dxdy$ is a valid iterated integral to write down.

(6) **(textbook 15.1.3)** Estimate $\int\int_R xe^{-xy} dA$, where $R = [0, 2] \times [0, 1]$ by using a Riemann sum with $m = n = 2$ and using upper-right corners. Then estimate it again using the midpoint rule.

(7) **(textbook 15.1.23)** Evaluate the double integral

$$\int_0^3 \int_0^{\pi/2} t^2 \sin^3 \phi d\phi dt.$$

(8) **(textbook 15.1.43)** Find the volume of the solid enclosed by the paraboloid $z = 2 + x^2 + (y - 2)^2$ and the planes $z = 1$, $x = 1$, $x = -1$, $y = 0$, and $y = 4$.

(9) **(textbook 15.1.49)** Evaluate the double integral

$$\iint_R \frac{xy}{1 + x^4} dA, \quad R = \{(x, y)| - 1 \leq x \leq 1, 0 \leq y \leq 1\}.$$

3. Notes

Original author: James Rowan.

All problems labeled "textbook" come from Stewart, James, *Multivariable Calculus: Math 53 at UC Berkeley*, 8th Edition, Cengage Learning, 2016.

Problems marked **(*)** are challenge problems, with problems marked **(**)** especially challenging problems.