MATH 53 DISCUSSION SECTION PROBLEMS – 3/23/23

1. Triple integrals in polar coordinates

(1) **(textbook 15.8.13)** Sketch the solid described by the inequalities $2 \le \rho \le 4$, $0 \le \phi \le \frac{\pi}{3}$, $0 \le \theta \le \pi$.

(2) **(textbook 15.8.41)** Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xydzdydx$.

(3) **(textbook 15.8.21)** Evaluate $\iiint_B (x^2+y^2+z^2)^2 dV$, where B is the ball with center the origin and radius 5.

(4) **(an old quiz)** Consider the solid region E bounded by the xy-plane and the paraboloid $z = 16 - x^2 - y^2$. What is the average height of a point in E above the xy-plane?

(5) **(an old quiz)** Using a triple integral, find the volume of the portion of the sphere of radius 2 centered at the origin lying between the cones $z = \sqrt{x^2+y^2}$ and $z = \sqrt{3x^2+3y^2}$ and above the xy-plane.

(6) **(an old quiz)** Using a triple integral, find the volume of the region lying above the cone $z = \sqrt{x^2+y^2}$ and below the surface $z = \sqrt{4-x^2-y^2}$.

(7) **(*)** What would an analogue of spherical polar coordinates for four-dimensional space look like? What would be the "hypervolume element" (i.e. the $dV = dxdydzdw$) be for spherical polar coordinates in four dimensions?

2. Notes

Original author: James Rowan.

All problems labeled "textbook" come from Stewart, James, *Multivariable Calculus: Math 53 at UC Berkeley*, 8th Edition, Cengage Learning, 2016.

Problems marked **(*)** are challenge problems, with problems marked **(**)** especially challenging problems.