MATH 53 DISCUSSION SECTION PROBLEMS – 3/21/23

1. Triple integrals

(1) True/false practice:
   (a) There is no way to interpret any triple integral using only three-dimensional geometry.

(2) **(textbook 15.6.9)** Evaluate the triple integral $\iiint_E y dV$, where $E = \{(x, y, z)|0 \le x \le 3, 0 \le y \le x, x - y \le z \le x + y\}$.

(3) **(textbook 15.6.21)** Use a triple integral to find the volume of the solid enclosed by the cylinder $y = x^2$ and the planes $z = 0$ and $y + z = 1$.

(4) **(textbook 15.6.36)** Write five other iterated integrals equal to $\int_0^1 \int_y^1 \int_0^z f(x, y, z) dxdzdy$.

(5) **(textbook 15.6.53)** Find the average value of the function $f(x, y, z) = xyz$ over the cube with side length $L > 0$ lying in the first octant with one vertex at the origin and three edges along the positive $x-$, $y-$, and z-axes.

2. Triple integrals in cylindrical coordinates

(6) True/false practice:
   (a) When we defined cylindrical coordinates, the choice of the xy-plane as the plane we expressed in polar coordinates was arbitrary; we could also have set up a polar coordinate system (r, θ, y), with (r, θ) describing the xz-plane in polar coordinates.

(7) **(textbook 15.7.29)** Evaluate $\int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^2 xz dzdxdy$ by changing to cylindrical coordinates.

(8) **(textbook 15.7.21)** Evaluate $\iiint_E x^2 dV$, where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$, and below the cone $z^2 = 4x^2 + 4y^2$.

(9) **(textbook 15.7.27)** Find the mass and center of mass of the solid S bounded by the paraboloid $z = 4x^2 + 4y^2$ and the plane $z = a$, $a > 0$ if S has a constant density K.

3. Notes

Original author: James Rowan.

All problems labeled "textbook" come from Stewart, James, *Multivariable Calculus: Math 53 at UC Berkeley*, 8th Edition, Cengage Learning, 2016.

Problems marked **(*)** are challenge problems, with problems marked **(**)** especially challenging problems.