MATH 53 DISCUSSION SECTION PROBLEMS – 3/16/23

1. Applications of double integrals

(1) **(textbook 15.4.7)** Consider a lamina occupying the region between $y = 0$ and $y = 1 - x^2$ with density $\rho(x, y) = ky$. Find the mass and center of mass of this lamina.

(2) **(requires triple integrals)** Holly, James, and Xiaohan are planning to meet at noon for tea. The random variables H, J, and X representing the number of hours it takes for them to show up are exponential random variables with mean 1 (i.e. all have probability density functions given by $f(x) = 0$ for $x < 0$, $f(x) = e^{-x}$ for $x \geq 0$). They agree to start when at least two of them show up, as long as it is not yet 1:00pm, at which time they will give up and not meet today. What is the probability that they actually meet for tea?

If you haven't read ahead/seen triple integrals yet, you can do this problem with just Holly and Xiaohan trying to meet and agreeing that one person will start by themselves if the other hasn't arrived yet, as long as it isn't 1pm yet.

(3) **(textbook 15.5.7)** Find the area of the part of the surface $z = y^2 - x^2$ lying between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

(4) Consider the triangle with vertices $(4, 0, 0)$, $(0, 4, 0)$, and $(0, 0, 4)$. Find the area of this triangle by using a double integral.

2. Notes

Original author: James Rowan.

All problems labeled "textbook" come from Stewart, James, *Multivariable Calculus: Math 53 at UC Berkeley*, 8th Edition, Cengage Learning, 2016.

Problems marked **(*)** are challenge problems, with problems marked **(**)** especially challenging problems.