MATH 53 DISCUSSION SECTION PROBLEMS – 2/9/23

1. Vector-valued functions: geometric aspects

(1) **(textbook 13.1.29)** Find three different surfaces that contain the curve $\mathbf{r}(t) = 2t\mathbf{i} + e^t\mathbf{j} + e^{2t}\mathbf{k}$. Sketch this curve.

(2) **(textbook 13.1.43)** Find a vector function representing the curve of intersection of the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1 + y$.

2. Arc length and curvature

(3) True/False practice:
   (a) If $\mathbf{r}(s)$ is a smooth vector function parametrized by arc length, then we know $|\mathbf{r}'(s)|$ at any value of s even without knowing anything else about $\mathbf{r}(s)$.
   (b) We defined curvature as $\frac{d\mathbf{T}}{ds}$, where s is arc length, purely for our convenience with computations and for no other reason.

(4) **(textbook 13.3.5)** Find the length of the curve $\mathbf{r}(t) = \langle 1, t^2, t^3 \rangle$ between $(1, 0, 0)$ and $(1, 1, 1)$.

(5) Reparametrize the curve $\mathbf{r}(t) = \langle 3t, 4t, 12t \rangle$ in terms of arc length from the point $(0, 0, 0)$.

(6) **(textbook 13.3.17)** Find the unit tangent vector to $\mathbf{r}(t) = \langle t, 3\cos t, 3\sin t \rangle$ and use the formula $\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$ to find the curvature of the curve.

3. Notes

Original author: James Rowan.

All problems labeled "textbook" come from Stewart, James, *Multivariable Calculus: Math 53 at UC Berkeley*, 8th Edition, Cengage Learning, 2016.

Problems marked **(*)** are challenge problems, with problems marked **(**)** especially challenging problems.