MATH 53 DISCUSSION SECTION PROBLEMS – 2/7/23

1. Lines and planes in 3D space

(1) True/false practice:
  (a) Two distinct lines in 3D space are either parallel or intersecting.
  (b) Three planes may all intersect in a point or not at all, but they can't all intersect in a line.
(2) **(textbook 12.5.71)** Find the distance from the point $(1, -2, 4)$ to the plane $3x + 2y + 6z = 5$.
(3) **(textbook 12.5.9)** Find parametric equations and symmetric equations for the line through the points $(-8, 1, 4)$ and $(3, -2, 4)$.
(4) **(textbook 12.5.29)** Find an equation of the plane through the point $\left(1, \frac{1}{2}, \frac{1}{3}\right)$ and parallel to the plane $x + y + z = 0$.
(5) **(textbook 12.5.33)** Find an equation of the plane through the points $(2, 1, 2)$, $(3, -8, 6)$, and $(-2, -3, 1)$.
(6) **(textbook 12.5.46)** Find the point of intersection of the line $x = t - 1$, $y = 1 + 2t$, $z = 3 - t$ and the plane $3x - y + 2z = 5$.
(7) **(textbook 12.5.29)** Are the planes $x + 4y - 3z = 1$ and $-3x + 6y + 7z = 0$ parallel, perpendicular, or neither? If they are neither, find the angle between them.
(8) **((*), compare to math 54)** How might we define the equation of a two-dimensional plane in 4-dimensional space with a vector equation? With symmetric equations? A line in 4D space? A general m-dimensional flat object in n dimensional space? How many symmetric equations do we need?

2. Vector-valued functions and space curves

(9) True/False practice:
  (a) Given a vector function $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$, its domain is the union of the domains of its three coordinate functions f, g, h.
  (b) If we have two curves $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ which intersect at a point (x_0, y_0, z_0) where neither $\mathbf{r}_1'(t)$ nor $\mathbf{r}_2'(t)$ are the zero vector, we can meaningfully define a notion of "the angle these two curves intersect at."
  (c) When we think about vector-valued functions as representing particle motion, it's important to remember the parametrization we're using.
(10) **(textbook 13.2.9)** Find the derivative of $\mathbf{r}(t) = \langle \sqrt{t-2}, 3, \frac{1}{t^2}\rangle$.
(11) **(textbook 13.2.17)** Find the unit tangent vector $\mathbf{T}(t)$ to $\mathbf{r}(t) = \langle t^2 - 2t, 1 + 3t, \frac{1}{3}t^3 + \frac{1}{2}t^2\rangle$ at $t = 2$.
(12) **(textbook 13.2.37)** Evaluate

$$\int_0^1 \left(\frac{1}{t+1}\mathbf{i} + \frac{1}{t^2+1}\mathbf{j} + \frac{t}{t^2+1}\mathbf{k}\right) dt$$

3. Notes

Original author: James Rowan.

All problems labeled "textbook" come from Stewart, James, *Multivariable Calculus: Math 53 at UC Berkeley*, 8th Edition, Cengage Learning, 2016.

Problems marked **(*)** are challenge problems, with problems marked **(**)** especially challenging problems.