MATH 53 DISCUSSION SECTION PROBLEMS – 2/23/23

1. Tangent planes

(1) True/false practice:
   (a) If f is a differentiable function of two variables, the vector $\langle f_x(a,b), f_y(a,b), 1\rangle$ is orthogonal to the tangent plane to the graph of $z = f(x,y)$ at the point $(a, b, f(a,b))$.
   (b) Every continuous function of two variables is differentiable.

(2) **(textbook 14.4.11)** Why is the function $f(x,y) = 1 + x\ln(xy-5)$ differentiable at the point $(2,3)$? What is its linearization $L(x,y)$ at that point?

(3) **(textbook 14.4.19)** Given that f is a differentiable function with $f(2,5) = 6$, $f_x(2,5) = 1$, and $f_y(2,5) = -1$, estimate $f(2.2, 4.9)$. Would you necessarily be able to use the same method to get a good estimate of $f(3,6)$?

(4) **(textbook 14.4.35)** A tin can is 8 cm in diameter, 12 cm high, and .04 cm thick. Estimate the volume of the tin in the can.

(5) **(an old exam)** Consider the surface $x^4 + y^4 + z^4 = 18$.
   (a) Find an equation of the plane tangent to the surface at the point $(2,1,1)$.
   (b) Give an approximation to $\sqrt[4]{18 - (1.01)^4 - (1.99)^4}$ and explain why it is a good approximation.

(6) **(*, textbook 14.4.43)** Show that $f(x,y) = x^2 + y^2$ is differentiable by finding values ϵ_1, ϵ_2 that go to 0 as $\Delta x, \Delta y$, respectively, go to zero such that $\Delta f = f_x(a,b)\Delta x + f_y(a,b)\Delta y + \epsilon_1\Delta x + \epsilon_2\Delta y$.

(7) **(*)** Prove that the tangent plane at the point (x_0, y_0, z_0) on the sphere $x^2 + y^2 + z^2 = 1$ has equation $x_0x + y_0y + z_0z = 1$ without using calculus.

2. Notes

Original author: James Rowan.

All problems labeled "textbook" come from Stewart, James, *Multivariable Calculus: Math 53 at UC Berkeley*, 8th Edition, Cengage Learning, 2016.

Problems marked **(*)** are challenge problems, with problems marked **(**)** especially challenging problems.