MATH 53 DISCUSSION SECTION PROBLEMS – 2/21/23

1. Partial derivatives

(1) True/False practice:
  (a) If f_{xyz} for $f(x, y, z) = g(x, y) + h(y, z) + k(x, z)$ for some sufficiently nice functions g, h, and k of two variables, we can easily see that $f_{xyz} = 0$.
  (b) If $u(x, y)$ is a solution of the partial differential equation $u_{xx} + u_{yy} = 0$, then for any four real numbers (a, b, c, d), the function $v(x, y) = u(x, y) + ax + by + cxy + d$ is also a solution of $u_{xx} + u_{yy} = 0$.

(2) **(textbook 14.3.17)** Find the first partial derivatives of $f(x, t) = t^2 e^{-x}$.

(3) **(textbook 14.3.53)** Find all the second partial derivatives of $f(x, y) = x^4 y - 2x^3 y^2$.

(4) **(textbook 14.3.71)** If $f(x, y, z) = xy^2 z^3 + \arcsin(x\sqrt{z})$, find f_{xyz}. *Hint: there's a way to make this less painful calculation-wise.*

(5) **(textbook 14.3.79)** If f and g are twice-differentiable functions of a single variable, show that the function

$$u(x, t) = f(x + at) + g(x - at)$$

is a solution of the *wave equation* $u_{tt} = a^2 u_{xx}$.

(6) Write down three functions of two variables satisfying Laplace's equation $u_{xx} + u_{yy} = 0$ and show that they are solutions.

2. Notes

Original author: James Rowan.

All problems labeled "textbook" come from Stewart, James, *Multivariable Calculus: Math 53 at UC Berkeley*, 8th Edition, Cengage Learning, 2016.

Problems marked **(*)** are challenge problems, with problems marked **(**)** especially challenging problems.