MATH 53 DISCUSSION SECTION PROBLEMS – 2/16/23

1. Limits of multivariable functions

(1) True/false practice:

(a) If $g(x, y, z)$ is a function of three variables whose domain is all of $\mathbb{R}^3$, then if we know that for some real number L,

$$\lim_{x\to 0} g(x, 0, 0) = \lim_{y\to 0} g(0, y, 0) = \lim_{z\to 0} g(0, 0, z) = L,$$

then

$$\lim_{(x,y,z)\to(0,0,0)} g(x, y, z) = L.$$

(2) **(textbook 14.2.5)** Find, if it exists, or explain why it doesn't if it doesn't:

$$\lim_{(x,y)\to(3,2)} (x^2y^3 - 4y^2).$$

(3) **(textbook 14.2.17)** Find, if it exists, or explain why it doesn't if it doesn't:

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}.$$

(4) **(textbook 14.2.21)** Find, if it exists, or explain why it doesn't if it doesn't:

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz^2 + xz^2}{x^2 + y^2 + z^4}.$$

(5) **(an old quiz)** Consider the function $h(x, y) = \frac{x^3-y^3}{x^3+y^3}$.

(a) What is the domain of this function? Where is this function continuous? Sketch the domain and the region where this function is continuous.

(b) Find, with justification, the limit

$$\lim_{(x,y)\to(2,1)} h(x, y),$$

if it exists, or explain why it doesn't if it doesn't.

(c) Find, with justification, the limit

$$\lim_{(x,y)\to(0,0)} h(x, y),$$

if it exists, or explain why it doesn't if it doesn't.

(6) **(**)** Prove or find a counterexample: for any function $f(x, y)$ defined on a subset of $\mathbb{R}^2$, if $\lim_{(x,y)\to(0,0)} f(x, y)$ exists along any path (i.e. parametric curve) going to the origin lying in the domain of f, then the limit exists.

2. Notes

Original author: James Rowan.

All problems labeled "textbook" come from Stewart, James, *Multivariable Calculus: Math 53 at UC Berkeley*, 8th Edition, Cengage Learning, 2016.

Problems marked **(*)** are challenge problems, with problems marked **(**)** especially challenging problems.