MATH 53 REVIEW PROBLEMS – 2/14/23

1. Fall 2013 Auroux midterm 1A

(1) Find the area enclosed by a loop of the curve given by the polar equation $r = \sqrt{\sin 2\theta}$.

(2) (a) Find the area of the space triangle with vertices $P_0 = (2, 1, 0), P_1 = (1, 0, 1)$, and $P_2 = (2, -1, 1)$.
   (b) Find the equation of the plane containing the three points P_0, P_1, P_2.
   (c) Find the intersection of this plane with the line which is parallel to the vector $\overline{V} = \langle 1, 1, 1\rangle$ and passes through the point $S = (-1, 0, 0)$.

(3) (a) Let $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ be the position vector of a path. Give a simple intrinsic formula for $d/dt(\mathbf{r} \cdot \mathbf{r})$ in vector notation (not using coordinates).
   (b) Show that if $\mathbf{r}$ has constant length, then $\mathbf{r}$ and $\mathbf{v} = \mathbf{r}'$ are perpendicular.
   (c) Let $\mathbf{a} = \mathbf{r}''$ be the acceleration. Still assuming that $\mathbf{r}$ has constant length, and using vector differentiation, express the quantity $\mathbf{r} \cdot \mathbf{a}$ in terms of the velocity vector only.

2. Fall 2014 Agol midterm

(2) Decide if the triangle with vertices

$$P(0, -3, -4), Q(1, -5, -1), R(5, -6, -3)$$

is right-angled:
   (a) using angles between vectors, and
   (b) using distances and the Pythagorean theorem.

(3) Find an equation for the plane that passes through the point $(-2, 4, -3)$ and is perpendicular to the planes $-x + 3y - 5z = 42$ and $y - 2z = -5$.

(4) Let $\mathbf{r}(t) = \langle \sin t, 2\cos t\rangle$.
   (a) Sketch the plane curve with the given vector equation.
   (c) Sketch the position vector $\mathbf{r}(t)$ and the tangent vector $\mathbf{r}'(t)$ for the value $t = \pi/4$ (use the same graph as for (a).

3. Notes

Both of these exams are taken from the Files tab of bCourses.

Problem 1 on Agol's midterm is omitted because it's the same as problem 1 on Auroux's midterm.

The other problems on each of these midterms are from later chapters of the textbook, which are not covered on our midterm 1.