MATH 53 DISCUSSION SECTION PROBLEMS – 1/31/23

1. Vectors and the geometry of space

(1) True/False practice:
  (a) It doesn't matter at all how you draw your coordinate axes in 3D.
  (b) If $\mathbf{v}$ is a vector, there is no such thing as $-\mathbf{v}$.

(2) **(textbook 12.1.15)** Find an equation of the sphere that passes through the point $(4, 3, -1)$ and has center $(3, 8, 1)$.

(3) **(textbook 12.2.15)** Find the sum of the vectors $\langle -1, 4\rangle$ and $\langle 6, -2\rangle$ and illustrate geometrically.

(4) **(textbook 12.2.30)** If a child pulls a sled through the snow on a level path with a force of 50 N exerted at an angle of 38 degrees above the horizontal, find the horizontal and vertical components of the force.

(5) **((*), classical)** How could we interpret the space of continuous functions on $[0, 1]$ as a vector space?

2. The dot product

(6) True/False practice:
  (a) If $\mathbf{v} \cdot \mathbf{w} > 0$, then the angle between $\mathbf{v}$ and $\mathbf{w}$ is acute.

(7) **(textbook 12.3.9)** Find $\mathbf{a} \cdot \mathbf{b}$ if $|\mathbf{a}| = 7$, $|\mathbf{b}| = 4$, and the angle between the two vectors is 30 degrees.

(8) **(textbook 12.3.28)** Find two unit vectors that make an angle of 60 degrees with $\mathbf{v} = \langle 3, 4\rangle$.

(9) **(textbook 12.3.29)** Find the acute angle between the lines $2x - y = 3$ and $3x + y = 7$. *Hint: can you find a vector parallel to each of these lines?*

(10) **(textbook 12.3.43)** Find the scalar and vector projections of $2\vec{i} + 4\vec{j} - \vec{k}$ onto $3\vec{i} - 3\vec{j} + \vec{k}$.

(11) **(**)** Consider the set $C([0, 1])$ of continuous real-valued functions defined on the unit interval $[0, 1]$. Show that the map taking two functions in $C([0, 1])$ to real numbers given by $f \bullet g = \int_0^1 f(x)g(x)dx$ satisfies the commutativity, distributivity over addition, and associativity with scalar multiplication properties from class, and that $f \bullet f \geq 0$ for all $f \in C([0, 1])$, with $f \bullet f = 0$ if and only if f is the constant function equal to 0. Now use it to define notions of "length" and "angle between" for continuous functions on $[0, 1]$. If you've seen Fourier series, is there a connection between this dot product and Fourier coefficients?

3. Notes

Original author: James Rowan.

All problems labeled "textbook" come from Stewart, James, *Multivariable Calculus: Math 53 at UC Berkeley*, 8th Edition, Cengage Learning, 2016.

Problems marked **(*)** are challenge problems, with problems marked **(**)** especially challenging problems.