MATH 53 DISCUSSION SECTION PROBLEMS – 1/24/23

1. Calculus with parametric curves

(1) True/False practice:
   (a) If we have a parametric curve and a time t such that $\frac{dy}{dt} = \frac{dx}{dt}$ at that time t, then at the corresponding point $\big(x(t), y(t)\big)$, the tangent line to the curve is parallel to the line $y = x$.
   (b) The areas inside the parametric curves $x_1(t) = \cos t, y_1(t) = \sin t, 0 \leq t \leq 2\pi$ and $x_2(t) = \cos(2t), y_2(t) = \sin(2t), 0 \leq t \leq \pi$ are the same.

(2) **(textbook 10.2.9)** Find an equation of the tangent line to the parametric curve $x = t^2 - t$, $y = t^2 + t + 1$ at the point $(0, 3)$.

(3) **(textbook 10.2.11)** Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the parametric curve $x = t^2 + 1$, $y = t^2 + t$. For which values of t is the curve concave upward?

(4) **(textbook 10.2.27)** Find the slope of the tangent line to the trochoid $x = r\theta - d\sin\theta$, $y = r - d\cos\theta$ in terms of θ. Show that if $d < r$, then the trochoid does not have a vertical tangent.

(5) **(textbook 10.2.33)** Find the area enclosed by the x-axis and the curve $x = t^3 + 1$, $y = 2t - t^2$.

(6) **(textbook 10.2.37)** Set up, *but do not evaluate*, an integral that represents the length of the curve $x = t + e^{-t}$, $y = t - e^{-t}$, $0 \leq t \leq 2$.

(7) **(*)** In the written homework we show that the area of the ellipse with Cartesian equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab by using calculus with parametric curves. Can you derive this formula without using calculus?

2. Polar coordinates

(8) True/False practice:
   (a) The points with polar coordinates (r, θ) given by $(2, \frac{\pi}{2})$ and $(-2, \frac{5\pi}{2})$ are the same point.
   (b) The curves $r = 3$ and $r = 6\cos\theta$ are both circles with the same radius.

(9) **(textbook 10.3.1a)** Plot the point with polar coordinates $(1, \pi/4)$ and find two other pairs of polar coordinates of this point, one with $r > 0$ and one with $r < 0$.

(10) **(textbook 10.3.9)** Sketch the region in the plane consisting of points whose polar coordinates satisfy $r \geq 0$, $\pi/4 \leq \theta \leq 3\pi/4$.

(11) **(textbook 10.3.27)** For each of the following curves, decide whether to represent it in polar or Cartesian coordinates (pick whichever you think is easiest), and give a representation in that coordinate system:
   (a) A line through the origin making an angle of $\frac{\pi}{6}$ with the positive x-axis.
   (b) A vertical line through the point $(3, 3)$.

3. Notes

Original author: James Rowan.

All problems labeled "textbook" come from Stewart, James, *Multivariable Calculus: Math 53 at UC Berkeley*, 8th Edition, Cengage Learning, 2016.

Problems marked **(*)** are challenge problems, with problems marked **(**)** especially challenging problems.