MATH 53 DISCUSSION SECTION PROBLEMS – 1/19/23

1. Parametric equations

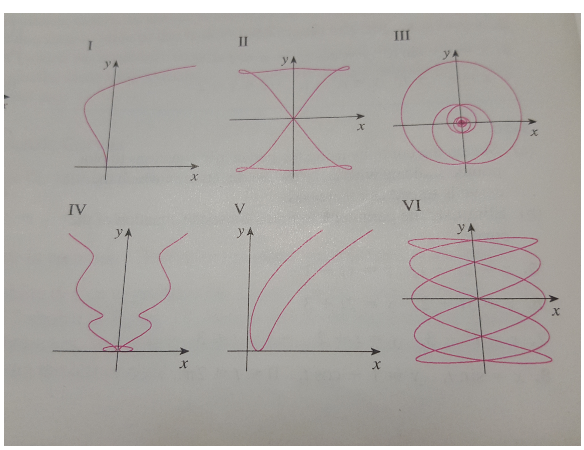
(1) True/False practice:
  (a) Given a curve, there are infinitely many different ways to parametrize it.
  (b) The parametric curves
$$x = \sin t, y = \sin^2 t, \quad \text{(no restrictions on t)}$$
  and
$$x = t, y = t^2, \quad \text{(no restrictions on t)}$$
  are the same curve.

(2) **(textbook 10.1.9)** Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases. Then eliminate the parameter to find a Cartesian equation of the curve: $x = t^2 - 3$, $y = t + 2$, $-3 \leq t \leq 3$.

(3) **(textook 10.1.13)** Eliminate the parameter to find a Cartesian equation of the curve. Then sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases: $x = \sin t$, $y = \csc t$, $0 < t < \pi/2$.

(4) **(textbook 10.1.33)** Find parametric equations for the path of a particle that moves along the circle $x^2 + (y - 1)^2 = 4$ in the manner described.
  (a) Once around the circle clockwise, starting at $(2, 1)$.
  (b) Three times around counterclockwise, starting at $(2, 1)$.
  (c) Halfway around counterclockwise, starting at $(0, 3)$.

(5) **(textbook 10.1.28)** Match the parametric equations with the graphs labeled I-VI (without using a graphing device). Give reasons for your choices.
  (a) $x = t^4 - t + 1$, $y = t^2$
  (b) $x = t^2 - 2t$, $y = \sqrt{t}$
  (c) $x = \sin(2t)$, $y = \sin(t + \sin 2t)$
  (d) $x = \cos(5t)$, $y = \sin(2t)$
  (e) $x = t + \sin(4t)$, $y = t^2 + \cos(3t)$
  (f) $x = \frac{\sin(2t)}{4+t^2}$, $y = \frac{\cos(2t)}{4+t^2}$



(6) **(*, classical)** Consider a circle of radius 3 and a circle of radius 2 tangent to this circle from the outside. We place a pen at the point where the two circles are tangent, then rotate the outer circle around the inner circle so that the outer circle stays tangent to the inner circle and the pen rotates as the outer circle rotates.
  (a) Sketch a rough picture of the situation.
  (b) Find parametric equations for the curve, called an *epicycloid*, that is traced by the pen.
  (c) How long does it take for the pen to be back at its initial position?
  (d) What happens if the radius of the inner circle is 5?
  (e) **(**)** What happens if the radius of the outer circle is $\sqrt{2}$? Supposing we kept rotating the outer circle around the inner circle forever, what would the graph that we drew look like?

2. Notes

Original author: James Rowan.

All problems labeled "textbook" come from Stewart, James, *Multivariable Calculus: Math 53 at UC Berkeley*, 8th Edition, Cengage Learning, 2016.

Problems marked **(*)** are challenge problems, with problems marked **(**)** especially challenging problems.