# MATH 53 DISCUSSION SECTION PROBLEMS – 1/19/23

## 1. Parametric equations

(1) (a) **True**: you can trace the curve out at any (constant or varying) speed you like.
  (b) **False**: The second is a parabola, and the first is only part of a parabola (because $-1 \le \sin t \le 1$ for all $t$).

(2) We can solve for $x$ as a function of $y$ by noticing that $t = y - 2$, so
$$x = (y-2)^2 - 3 = (y^2 - 4y + 4) - 3 = y^2 - 4y + 1.$$
The inequality $-3 \le t \le 3$ is equivalent to $-1 \le y \le 5$, so we plot the function above for these values of $y$.
Picture to be added later.

(3) Since $\csc t = \frac{1}{\sin t}$, this is (part of) the hyperbola $y = 1/x$. Which part is it? Since $0 < t < \pi/2$, we have $0 < \sin t < 1$, so it's the part between $x = 0$ and $x = 1$, traced from the vertical asymptote at $x = 0$, $y \to +\infty$ to the point $(1, 1)$.

(4) Some (not the only) possible answers:
  (a) $x = 2\cos t, y = 1 - 2\sin t, 0 \le t \le 2\pi$
  (b) $x = 2\cos t, y = 1 + 2\sin t, 0 \le t \le 6\pi$
  (c) $x = 2\cos t, y = 1 + 2\sin t, \pi/2 \le t \le 3\pi/2$

(5) (a) V: it's the only one where $x$ and $y$ are always nonnegative.
  (b) I: $y$ starts at 0 and only gets positive; $x$ is briefly negative and then positive
  (c) $x = \sin(2t)$, $y = \sin(t + \sin(2t))$
  II: both $x$ and $y$ oscillate between $-1$ and 1, but not as regularly as in VI.
  (d) VI: this is one of the Lissajous figures described in the textbook; both $x$ and $y$ oscillate between $-1$ and 1.
  (e) IV: it roughly follows the parabola $x = t, y = t^2$, but with squiggles around it created by the sine and cosine.
  (f) III: when $t \approx 0$ it looks like the circle $(\frac{\sin 2t}{4}, \frac{\cos 2t}{4})$, but as $t$ becomes large, the denominator does too, and the curve spirals inwards towards the origin.

(6) Omitted.