

18.784: SEMINAR IN NUMBER THEORY
PROBLEM SET 4
DUE BY 11:59PM FRIDAY OCT 15

1. Write down the first five elements of the “coherent sequences” (in the sense of Krit’s presentation) associated to:

- (1) $2/7$ in \mathbf{Q}_3 ,
- (2) $1715/3$ in \mathbf{Q}_7 ,
- (3) A square root of -1 in \mathbf{Q}_5 .

2. Show that $p^n\mathbf{Z}_p$ is compact for every integer n . (For this problem, use as the definition of “compact” that every open cover has a finite subcover. You may use one of other definitions mentioned by Carl, provided that you state the equivalence of definitions correctly.)

3. (Problem 114 of Gouvêa.) Let $x \in \mathbf{Z}_p$. What condition on its p -adic expansion will guarantee that x is a p -adic unit? Explain with proof.

4. Rachana defined a bijection between the (middle thirds) Cantor set C and \mathbf{Z}_2 . (See Theorem 4.4.1 of the book.) Complete the proof that it is a homeomorphism.