**18.784: SEMINAR IN NUMBER THEORY**
**PROBLEM SET 1**
**DUE BY 11:59PM FRIDAY SEP 24**

**1.** Prove Lemma 2.2.1 from the book (3rd edition numbering).
**2.** Let us call the *distance* between two $p$-adic numbers $x, y$ the value

$$d_p(x, y) := |x - y|_p = p^{-v_p(x-y)}.$$

For $p = 3$, calculate all the mutual distances between the numbers in the set $S := \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ in $\mathbf{Q}_3$.

(1) Divide the elements of $S$ into groups as large as possible so that the distance between any two elements within each group is at most 1. What are the groups?

(2) Divide the elements of $S$ into groups as large as possible so that the distance between any two elements within each group is at most $1/3$. What are the groups?

(3) Divide the elements of $S$ into groups as large as possible so that the distance between any two elements within each group is at most $1/9$. What are the groups?