# Nakajima Problem Set 2

**Exercise 5.** (1) In terms of $T$-weights on tangent spaces $T_pM$ at various fixed points $p \in M^T$, describe the hyperplanes.

(2) Show that the chamber structure for $M = T^*(\text{flag variety})$ is identified with usual Weyl chambers.

(3) Show that the chamber structure for $M = \widetilde{\mathcal{U}}_r^d$ is identified with the usual Weyl chambers for $\mathrm{SL}(r)$.

(4) Compute the chamber structure for $M = \widetilde{\mathcal{U}}_r^d$, but with the larger torus $T \times \mathbb{C}^\times_{\mathrm{hyp}}$.

**Exercise 6.** (1) Determine $\mathcal{A}_M$ for $M = T^*(\text{flag variety})$.

(2) Determine $\mathcal{A}_M$ for $M = \widetilde{\mathcal{U}}_r^d$ with respect to $T \times \mathbb{C}^\times_{\mathrm{hyp}}$.

**Exercise 7.** See §4(i) for the definition of the nearby cycle functor.

Suppose that the followings are given: two families $f_{\mathcal{X}} \colon \mathcal{X} \to \mathbb{C}$, $f_{\mathcal{M}} \colon \mathcal{M} \to \mathbb{C}$, together with a proper morphism $\Pi \colon \mathcal{M} \to \mathcal{X}$ such that $f_{\mathcal{M}} = f_{\mathcal{X}} \circ \Pi$.

(1) Check

$$\psi_{f_X} \Pi_* = \pi_* \psi_{f_M}$$

from the base change.

(2) Suppose that $f_{\mathcal{M}} \colon \mathcal{M} \to \mathbb{C}$ is a topologically trivial fibration, i.e., there is a homeomorphism $\mathcal{M} \to M \times \mathbb{C}$ such that $f_{\mathcal{M}}$ is sent to the projection $M \times \mathbb{C} \to \mathbb{C}$. Check

$$\psi_{f_M} \mathcal{C}_{\mathcal{M}} = \mathcal{C}_M[1].$$

**Exercise 8.** Give a proof of Proposition 4.7, i.e., We have a natural isomorphism

$$H_*(Z_{\mathcal{A}}) \cong \mathrm{Ext}^*_{D^b(X_0^T)}(\pi_*^T(\mathcal{C}_{M^T}), p_* j^! \pi_*(\mathcal{C}_M)).$$