Exercise 1. (a) We define the factorization morphism π^d for $G = \mathrm{SL}(r)$ in terms of (B_1, B_2, I, J). Let $\pi^d([B_1, B_2, I, J]) \in S^d\mathbb{C}$ be the spectrum of B_1 counted with multiplicities. Check that π^d satisfies the properties (1),(2):

(1) On the factor $S^{d-d'}(\mathbb{C}^2)$, it is given by the projection $\mathbb{C}^2 \to \mathbb{C}^1$, $(z_1, z_2) \mapsto z_1$.

(2) Consider $C_1 + C_2 \in S^d\mathbb{C}^1$ such that $C_1 \in S^{d_1}\mathbb{C}^1$, $C_2 \in S^{d_2}\mathbb{C}^1$ are disjoint. Then $(\pi^d)^{-1}(C_1 + C_2)$ is isomorphic to $(\pi^{d_1})^{-1}(C_1) \times (\pi^{d_2})^{-1}(C_2)$.

(b) Check that $\mathcal{F}|_{\mathbb{P}^1_x}$ is trivial if $B_1 - x$ is invertible.

More generally one can define the projection as the spectrum of $a_1B_1 + a_2B_2$ for $(a_1, a_2) \in \mathbb{C}^2 \setminus \{0\}$, but it is enough to check this case after a rotation by the GL(2)-action.

Exercise 2. [Nak99, Remark 8.19] Define operators $P^\Delta_{\pm 1}(\alpha)$ acting on $\bigoplus_n H^*(S^nX)$ for a (compact) manifold X in a similar way, and check the commutation relation (1) with $r = 1$:

$$\left[P^\Delta_m(\alpha), P^\Delta_n(\beta)\right] = rm\delta_{m,-n}\langle\alpha, \beta\rangle\,\mathrm{id}\,. \tag{1}$$

Exercise 3. Let $\mathrm{Gr}(d, r)$ be the Grassmannian of d-dimensional subspaces in $\mathbb{C}^r$, where $0 \le d \le r$. Let $M = T^*\,\mathrm{Gr}(d, r)$. Determine $X = \mathrm{Spec}(\mathbb{C}[M])$. Study fibers of the affinization morphism $\pi\colon M \to X$ and show that π is semi-small. Compute graded dimensions of IH^* of strata, using the well-known computation of Betti numbers of $T^*\,\mathrm{Gr}(d, R)$.

Exercise 4. Show the assertion that the Heisenberg algebra acts trivially on the first factor $\bigoplus_d IH^{[*]}_{\mathbb{G}}(\mathrm{Bun}^d_{\mathrm{SL}(r)})$.