MATH 274 SCHEDULE

TONY FENG

This is a tentative week-by-week schedule for Math 274: Topics in Algebra.

The material is subject to change. The dependency of material between different Parts will be essentially nonexistent. The dependency of material between different weeks within the same Part will be weak (mostly motivational), but there will be some dependency for Parts 3,4, and 5.

Usually, weeks will begin with a non-technical survey-style lecture (similar to a research seminar talk). For such lectures, the date is bolded. Some talks will be devoted to background, and in this case the date is italicized.

Bold = non-technical lecture

Italicized = background lecture

The following weeks are fair game for scribes: 3, 4, 6, 7, 8, 9, 10, 11, 12.

CONTENTS

Part 1. **Global and local Langlands correspondences**

WEEK 1: COHOMOLOGY OF ARITHMETIC GROUPS

Jan 22. Introduction and cohomology of arithmetic groups.
Jan 24. The work of Treumann–Venkatesh.

- References: 1407.2346.
- Topics: Smith theory, reductive groups, Hecke algebras, Satake isomorphism

WEEK 2: GEOMETRIZATION OF THE LOCAL LANGLANDS CORRESPONDENCE

Jan 27. The Geometric Satake Equivalence.

- References: 0401222, 1603.05593
- Topics: affine Grassmannians, perverse sheaves, Tannaka duality

Jan 29. Introduction to Geometric Langlands.

- References: Informal Introduction to Geometric Langlands.
- Topics: Geometric Satake, Bun_G, moduli of shtukas, V. Lafforgue's construction of L-parameters.

Jan 31. The Local Langlands correspondence and the work of Fargues–Scholze.

- References: Gross' Eilenberg lectures, 2102.13459.
- Perfectoid spaces, Fargues–Fontaine curve, Local Langlands correspondence and its geometrization.

WEEK 3: MODULAR FUNCTORIALITY IN THE LOCAL LANGLANDS CORRESPONDENCE

Feb 3. The Treumann–Venkatesh Conjectures.

- References: 2312.12542.
- Topics: reductive groups, Local Langlands correspondence, Langlands functoriality.

Feb 5. Smith–Treumann localization.

- References: 1107.3798, 1708.08174.
- Topics: Smith theory, 6-functor formalism.

Feb 7. L-parameters and Tate cohomology.

- References: 2105.06341, 2312.12542.
- Topics: Tate cohomology, Deligne–Lusztig theory, supercuspidal representations.

Part 2. **Algebraic** K-**theory**

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WEEK 4: THE GALOIS ACTION ON SYMPLECTIC K-THEORY

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Feb 10. The Galois action on stable homology of symplectic groups.
- References: 2007.15078

Feb 12. Complex multiplication of abelian varieties.
- References: 2007.15078.

Feb 14. Introduction to symplectic K-theory.
- References: 2007.15078
- Topics: algebraic K-theory, Bloch-Kato conjectures.

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WEEK 5: ALGEBRAIC K-THEORY

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Feb 17. No class.
Feb 19. The K-theory of finite fields I.
- References: On the Cohomology and K-Theory of the General Linear Groups Over a Finite Field.
- Topics: Plus construction, overview of Quillen's computation.

Feb 21. The K-theory of finite fields II.
- References: On the Cohomology and K-Theory of the General Linear Groups Over a Finite Field.
- Topics: Brauer lift, group cohomology.

Part 3. **Steenrod operations and applications to Brauer groups**

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WEEK 6: ÉTALE STEENROD OPERATIONS AND THE ARTIN–TATE PAIRING

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Feb 24. The Artin–Tate Conjecture.
- References: On the conjectures of Birch and Swinnerton-Dyer and a geometric analog.
- Topics: Brauer groups, Artin–Tate dictionary

Feb 26. The Artin–Tate pairing I.
- References: 1706.00151.
- Topics: the Artin–Tate pairing, Steenrod operations.

Feb 28. The Artin–Tate pairing II.
- References: 1706.00151.
- Topics: Wu's formula, étale homotopy theory.

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WEEK 7: PRISMATIC STEENROD OPERATIONS

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Mar 3. The Milne–Artin–Tate pairing.
- References: TBA.
- Topics: syntomic cohomology, motivic cohomology, Bhatt–Morrow–Scholze.

Mar 5. Overview of prismatic cohomology
- References: 1905.08229.

Mar 7. Prismatization and prismatic Steenrod operations.
- References: Prismatic F-gauges
- Topics: Drinfeld/Bhatt–Lurie stacks.

Part 4. **The Breuil–Mézard Conjecture**

Week 8: Mirror symmetry and the Breuil–Mezard Conjecture

Mar 10. The Emerton–Gee stack and the Breuil–Mézard Conjecture.
- References: 1907.07185, 2009.06708
- Topics: (families of) local Galois representations, p-adic Hodge theory

Mar 12. Construction of Breuil–Mézard cycles I.
- References: 2310.07006.
- Topics: local models for moduli of Galois representations, affine Springer fibers, mirror symmetry, moduli of Higgs bundles.

Mar 14. Construction of Breuil–Mézard cycles II.
- References: 2310.07006.
- Topics: modular representation theory, Beilinson-Bernstein localization, BMR localization

Week 9: Mirror symmetry and the Breuil–Mezard Conjecture over ramified fields

Mar 17. Equivariant homology of affine Springer fibers.
- References: 0305144

Mar 19. Verification of Breuil–Mezard relations.
- References: 2310.07006.
- Topics: Weyl character formula, equivariant localization

Mar 21. Generalization to ramified fields.
- References: TBA.
- Topics: ramified local models.

SPRING BREAK

Part 5. **Higher theta functions**

Week 10: Introduction to higher theta functions

Mar 31. Classical and arithmetic theta functions.
- References: 0308295

April 2. No class.

April 4. Moduli of unitary shtukas and their special cycles.
- References: 2103.11514, 2110.07001.

Week 11: Higher Siegel–Weil formulas

April 7. Derived fundamental classes.
- References: 1909.01332, 2409.03035
- Topics: simplicial commutative rings, derived algebraic geometry

April 9. The higher Siegel–Weil formula.
- References: 1909.01332
- Topics: Hitchin spaces, Hitchin fibration, perverse sheaves

April 11. Proof of the higher Siegel–Weil formula.

- References: 1909.01332
- Topics: perverse sheaves, Springer theory, Fourier coefficients of Eisenstein series, local density polynomials.

WEEK 12: MODULARITY OF HIGHER THETA FUNCTIONS

April 14. The Modularity Conjecture.
- References: 2308.10979, 2403.19711.
- Topics: cohomological correspondences, motivic sheaves.

April 16. The case $r = 0$.
- References: 2308.10979.

April 18. Derived Fourier analysis.
- References: 2308.10979, 2403.19711.
- Topics: derived vector bundles, Deligne–Laumon Fourier transform.

WEEK 13: GEOMETRIZATION OF THE LOCAL THETA CORRESPONDENCE

April 21. The theta sheaf.
- References: 0405021, 2102.13459.
- Topics: theta functions, Weil representation, Hecke eigensheaves, Banach–Colmez spaces.

April 23. Theta-sheaf and universal Weil representation.
- References: TBA.

April 25. Theta-correspondence and Local Langlands functoriality.
- References: TBA.

Part 6. **Student presentations**

WEEK 14: STUDENT PRESENTATIONS

April 28: TBA
April 30: TBA
May 2: TBA

WEEK 15: READING WEEK

Student presentations continue as necessary.