Recent Developments in Weak Convergence Methods for Nonlinear PDE

SIAM Short Course
Boston, July 9, 2006

Lawrence C. Evans
Department of Mathematics
University of California, Berkeley

In memory of Ron DiPerna, *il miglior fabbro.*

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