Errata for "Measure Theory and Fine Properties of Functions, Revised Edition"
by L. C. Evans and R. F. Gariepy
CRC Press, 2015

Last modified: August, 2019.

A huge number of typos appeared in the revised edition:

CHAPTER 1

page 12, line 1: Change to $\lim_{m\to\infty}$
page 18, lines -8 and -10: Change $[-\infty, a]$ to $[-\infty, a)$
page 21, line 9: Change $f(x)$ to $f(a)$
page 21, line 10: Should read "...can be measure..."
page 21, line -7: Change to $\{B_{ij}\}_{j=1}^{\infty}$
page 22, line 8: Should be K_{i1}
page 23, line 6: Should read "...with respect to the measure..."
page 36, line 14: Change to $\frac{D}{2^{j-1}}$
page 40, line 2: Should be $B(a, r)$
page 42, line -2: Change to "$\dots \cos\theta - |a_i|^2 \dots$"
page 45, line 9: "Borel" should be "Radon"
page 51, line -3: Change to $\int_A$
page 55, line -3: Change to $\lim_{r\to 0}$
page 63, line 11: Remove extra)
page 64, line -2: Remove extra)
page 65, line -3: Change lim sup to $\limsup_{k\to\infty}$
page 66, line 6: Change to $\mu(B(R))$
page 74, line 14: Should be $\sup_{j,l_j>m_1}\{\phi_{k_j}(m_1) - \phi_{k_j}(l_j)\}$

CHAPTER 2

page 85, line -2: H should be $\mathcal{H}$
page 89, line -8: Change to "Lemma 2.3"
page 90, line -5: Change to "Theorem 2.3"
page 91, line 8 and line -8: H should be $\mathcal{H}$
page 93, line -8: H should be $\mathcal{H}$
page 94, line -3: H should be $\mathcal{H}$
page 95, line 3: H should be $\mathcal{H}$
page 96, line 2: Delete "$\leq \mathcal{H}^s_\infty(C \cap E)$"
page 96, line 6: H should be $\mathcal{H}$
page 99, line 10: Change to $B(x, r)$
page 99, line -9: Change to Λ^ϵ_s

CHAPTER 3

page 101, line 12: Change to "Jacobian"
page 111, line 7: Should be L^*
page 115, lines 5 and 6: Add) to all expressions in numerators
page 116, line 4: Should be "$\leq \frac{1}{i}$"
page 135, line -4: Change to "Lemma 3.6"
page 136, line 5: Change to "Lemma 3.6"
page 119, line -7: Should be "Then"
page 123, line 3: Replace j with 1
page 131, line 12: Change to " $)\, dy$"
page 132, line -5: Add "|" before the period
page 139, line 10: Change to $\mathbb{R}^n$

CHAPTER 4

page 152, line 5: Change to $\bar{U}$ and delete the misplaced overbar
page 152, line -3: Delete the comma after Q
page 157, line -2: Should be $|\beta'_\epsilon(f)Df|$
page 164, lines 4 and 5: Change to $\mathbb{R}^n$
page 165, line 9: Should be =; larger font for domain of integration
page 166, line -8: Should be g
page 169, line 6: Change $f_k(x)$ to $\bar{f}_k(x)$
page 170, line 2: Change to $||$
page 170, line 5: Change $||\,|$ to $||$
page 170, line 7: Change to $\bar{f}_{k_i}$
page 173, line -6: Change to "$\}^0$"
page 174, line -1: Should be g_l
page 178, line 13: Should be $\frac{\epsilon}{2^m}$
page 180, line -3: Should be $\leq \frac{C}{S_j^p}$
page 181, line 2: Change ∞ to n
page 182, line 9: Change to $\leq$
page 187, line-9: Change to $\fint_{B(x,r)}$
page 189, line 12: Change x to x'

CHAPTER 5

page 195, line -3: Change to $\bar{L}(\phi)$ and delete the misplaced overbar
page 197, line -5: Remove "f" from the formula for $||\partial E||$
page 201, line -1: Change to "$\zeta_k(\eta_{\epsilon_k} * \phi))$"
page 202, lines 1,2,3: Change all $|Df|$ to $||Df||$
page 202, line -3: Should be $\phi \cdot d\mu_k$
page 206, line 3: Should be $||Df||(C_{\epsilon,\delta})$

page 206, line 4: Change subscript to $\epsilon > 0$
page 207, lines -8 and -10: Change to $(f_k)_t$
page 208, lines -3: Should be $B(x,r) \cap U$
page 209, lines 5 and 10: H should be $\mathcal{H}$
page 209, lines -9: Add "Let γ give a local parameterization of ∂U near x."
page 209, line 6: Put σ in place of γ
page 211, line 1: Change to "...$f \in BV(U)$. Note that the set U..."
page 213, line 13: Less space after "div"
page 216: Add hypothesis that $n > 1$
page 217, line 11: Should be $B(x,r) \cap U$
page 219, line -2: Change 1^* to $1 - \frac{1}{n}$
page 221, line 8: Should be "verify"
page 226, line -1: Change to D_r
page 227, line 2: Should be $(E \cap B(rL))$
page 227, line -9: Change to $\mathbb{R}^n$
pages 227-229: This proof contains an error; substitute the correction at the end of this document.
page 230, line 5: Add comma
page 234, line -4: Change (ii) to (iii)
page 236, lines 12 and 14: Interchange "Lemma 5.2" and "Lemma 5.5"
page 241, line -7. Interchange H_ν^+ and H_ν^- in formulas for $\mu(x)$, $\lambda(x)$.
page 242, lines 2 and 3: Add "$\lim_{r \to 0}$"
page 242, line 4: Should be "$0 < r < 1$"
page 243, line 3 : Change to $\{f > M\}$
page 243, lines 3 and 9: Change $(f - M)$ to $(f - M)^+$
page 243, line 8: Should be Theorem 5.10
page 244, line 7: Change to $(\star\star\star\star)$
page 246, line 6: Remove | after the 1
page 246, line 13: Should be $\int_c^d$
page 249, line 12: Change to C_c^1
page 252, line -9: Should be $B(z,r)$
page 253, line 7: Should be $(\star\star\star\star)$
page 253, line -10: Add space before f^z

CHAPTER 6

page 259, line -1: Remove extra)
page 260, line 1: Should be $|[Df]_s|$
page 261, line 9: Change $B(r)$ to $B(sr)$
page 261, line -2: Delete ",(ii)"
page 263, line 8: Remove "$\cdot$"

page 263, line 10: Change to "=: a"
page 264, line 5: Delete (before $1 \leq p$...
page 269, line -9: Change to $|y - z|$
page 269, line -3: Change to $B(x, r)$
page 269, line -2: Change to (ii)
page 270, line -11: Should be (ii)
page 270, line -10: Should be (iii)
page 272, line 5: Move qed box to end of proof
page 272, line 8: Add - before the first =
page 274, line 2: Should be f^ϵ
page 275, line -3: Change to ess sup
page 276, line -9: Change to ess sup
page 284, line -3. Change to $B(x, \frac{r}{2^{k+1}})$
page 287, line -10. Change to $W^{1,p}(\mathbb{R}^n)$
page 288, line 7. Change 6.12 to 6.11

I have been extremely slow in posting these errata that readers have found for the revised edition of our book. Many belated thanks to D Ferizovic, W Ozanski, A Rajapakse and M Safdari for sending me lengthly lists of typos, errors and useful comments.

Please let me know about any other mistakes you find, at evans @math.berkeley.edu.

See the next page for correction for pages 227-229.

Correction for pages 227-229

4. Claim #1: $\nu_F = e_n$ $\|\partial F\|$-a.e.

Proof of claim: First note that since $0 \in \partial^* E$ and $|\nu_E| = 1$ $\|\partial E\|$-a.e., we have

$$\lim_{r\to 0} \fint_{B(r)} |\nu_E - e_n|^2 \, d\|\partial E\| = 2 \lim_{r\to 0} \fint_{B(r)} 1 - \nu_E \cdot e_n \, d\|\partial E\| = 0. \qquad (\star\star)$$

Let us now write $\nu_j := \nu_{E_j}$. Then if $\phi \in C_c^1(\mathbb{R}^n; \mathbb{R}^n)$, we have

$$\int_{\mathbb{R}^n} \phi \cdot \nu_j \, d\|\partial E_j\| = \int_{E_j} \operatorname{div} \phi \, dy \quad (j = 1, 2, \dots).$$

Since

$$\chi_{E_j} \to \chi_F \quad \text{in } L^1_{\text{loc}},$$

we see from the above and $(\star)$ that

$$\int_{\mathbb{R}^n} \phi \cdot \nu_j \, d\|\partial E_j\| \to \int_{\mathbb{R}^n} \phi \cdot \nu_F \, d\|\partial F\|$$

as $j \to \infty$.

In addition, for all ϕ as above,

$$\int_{\mathbb{R}^n} \phi \cdot \nu_j \, d\|\partial E_j\| = \frac{1}{s_j^{n-1}} \int_{\mathbb{R}^n} (\phi \circ g_{s_j}) \cdot \nu_E \, d\|\partial E\|;$$

consequently for $r > 0$:

$$\begin{cases} \|\partial E_j\|(B(r)) = \frac{1}{s_j^{n-1}} \|\partial E\|(B(s_j r)) \\ \int_{B(r)} \nu_j \, d\|\partial E_j\| = \frac{1}{s_j^{n-1}} \int_{B(s_j r)} \nu_E \, d\|\partial E\|. \end{cases}$$

So $(\star\star)$ implies

$$\fint_{B(r)} |\nu_j - e_n|^2 \, d\|\partial E_j\| = \fint_{B(s_j r)} |\nu_E - e_n|^2 \, d\|\partial E\| \to 0,$$

as $j \to \infty$. Select $\zeta \in C_c^1(\mathbb{R}^n)$ such that $\zeta \geq 0$, and put $\phi = \zeta e_n$ above. Then

$$\int_{\mathbb{R}^n} \zeta e_n \cdot \nu_F \, d\|\partial F\| = \lim_{j\to\infty} \int_{\mathbb{R}^n} \zeta e_n \cdot \nu_j \, d\|\partial E_j\| = \lim_{j\to\infty} \int_{\mathbb{R}^n} \zeta \, d\|\partial E_j\|. \qquad (\star\star\star)$$

Choose now any radius $r > 0$ for which $\|\partial F\|(B(r)) = 0$. Pick $h > 0$ and select the function ζ above so that $0 \leq \zeta \leq 1$, $\zeta \equiv 1$ on $B(r)$, $\zeta \equiv 0$ on $\mathbb{R}^n - B(r+h)$. Then lower semicontinuity and $(\star\star\star)$ imply

$$\|\partial F\|(B(r)) \leq \int_{B(r+h)} \zeta e_n \cdot \nu_F \, d\|\partial F\|.$$

Sending $h \to 0$, we find that

$$\|\partial F\|(B(r)) \leq \int_{B(r)} e_n \cdot \nu_F \, d\|\partial F\|.$$

for all r as above. Since $e_n \cdot \nu_F \ \leq 1$, it follows that $e_n \cdot \nu_F = 1$ $\|\partial F\|$-a.e. and so the claim holds.

We also see from the above that

$$\|\partial F\|(B(r)) = \lim_{j \to \infty} \|\partial E_j\|(B(r))$$

whenever $\|\partial F\|(\partial B(r)) = 0$.