

**Errata for “Measure Theory and Fine Properties of  
Functions, Revised Edition”  
by L. C. Evans and R. F. Gariepy  
CRC Press, 2015**

Last modified: May 6, 2024.

A huge number of typos have unfortunately appeared in the revised edition. We will correct these in the forthcoming new printing this year.

**CHAPTER 1**

- page 4, lines 3 and 5: Change  $\mathbb{R}^n$  to  $X$
- page 8, lines 3-7: This proof is incomplete: see the correction below.
- page 12, line 1: Change to  $\lim_{m \rightarrow \infty}$
- page 17, Theorem 1.11 For (i) add the assumption that  $\mu\{f = \pm\infty\} = \mu\{g = \pm\infty\} = 0$ . For (ii) add the assumption that  $\mu\{f_k = \pm\infty\} = 0$ .
- page 18, lines -8 and -10: Change  $[-\infty, a]$  to  $[-\infty, a)$
- page 21, line 9: Change  $f(x)$  to  $f(a)$
- page 21, line 10: Should read “...can be measure...”
- page 21, line -7: Change to  $\{B_{ij}\}_{j=1}^{\infty}$
- page 22, line 8: Should be  $K_{i1}$
- page 23, line 6: Should read “...with respect to the measure...”
- page 31, line -10: Change to “ $\mu$ -integrable for  $\nu$  a.e.  $y \in Y$ ...”
- page 36, line 14: Change to  $\frac{D}{2^j-1}$
- page 40, line 2: Should be  $B(a, r)$
- page 42, line -2: Change to “...  $\cos \theta - |a_i|^2$ ...”
- page 44, line 3: Should be “ $a_i - a_k$ ”
- page 45, line 9: “Borel” should be “Radon”
- page 51, line -3: Change to  $\int_A$
- page 55, line -3: Change to  $\lim_{r \rightarrow 0}$
- page 63, line 11: Remove extra )
- page 64, line -2: Remove extra )
- page 65, line -3: Change  $\lim \sup$  to  $\lim \sup_{k \rightarrow \infty}$
- page 66, line 6: Change to  $\mu(B(R))$
- page 74, line 14: Should be  $\sup_{j, l_j > m_1} \{\phi_{k_j}(m_1) - \phi_{k_j}(l_j)\}$

**CHAPTER 2**

- page 85, line -2:  $H$  should be  $\mathcal{H}$
- page 89, line -8: Change to “Lemma 2.3”
- page 90, line -5: Change to “Theorem 2.3”
- page 91, line 8 and line -8:  $H$  should be  $\mathcal{H}$
- page 93, line -8:  $H$  should be  $\mathcal{H}$

page 94, line -3:  $H$  should be  $\mathcal{H}$   
 page 95, line 3:  $H$  should be  $\mathcal{H}$   
 page 96, line 2: Delete " $\leq \mathcal{H}_\infty^s(C \cap E)$ "  
 page 96, line 6:  $H$  should be  $\mathcal{H}$   
 page 99, line 10: Change to  $B(x, r)$   
 page 99, line -9: Change to  $\Lambda_s^\epsilon$

### CHAPTER 3

page 101, line 12: Change to "Jacobian"  
 page 109, line -3: Change to " $O \circ O^* = I$  on  $O(\mathbb{R}^n) \subseteq \mathbb{R}^m$ "  
 page 111, line 7: Should be  $L^*$   
 page 115, lines 5 and 6: Add  $)$  to all expressions in numerators  
 page 116, line 4: Should be " $\leq \frac{1}{i}$ "  
 page 135, line -4: Change to "Lemma 3.6"  
 page 136, line 5: Change to "Lemma 3.6"  
 page 119, line -7: Should be "Then"  
 page 123, line 3: Replace  $j$  with 1  
 page 131, line 12: Change to " $) dy$ "  
 page 132, line -5: Add " $|$ " before the period  
 page 139, line 10: Change to  $\mathbb{R}^n$

### CHAPTER 4

page 152, line 5: Change to  $\bar{U}$  and delete the misplaced overbar  
 page 152, line -3: Delete the comma after  $Q$   
 page 157, line -2: Should be  $|\beta'_\epsilon(f)Df|$   
 page 164, lines 4 and 5: Change to  $\mathbb{R}^n$   
 page 165, line 9: Should be  $=$ ; larger font for domain of integration  
 page 166, line -8: Should be  $g$   
 page 169, line 6: Change  $f_k(x)$  to  $\bar{f}_k(x)$   
 page 170, line 2: Change to  $||$   
 page 170, line 5: Change  $|||$  to  $||$   
 page 170, line 7: Change to  $\bar{f}_{k_i}$   
 page 173, line -6: Should be  $g_i$   
 page 174, line -1: Change to " $\}^0$ "  
 page 178, line 13: Should be  $\frac{\epsilon}{2^m}$   
 page 180, line -3: Should be  $\leq \frac{C}{S_j^p}$   
 page 181, line 2: Change  $\infty$  to  $n$   
 page 182, line 9: Change to  $\leq$   
 page 187, line-9: Change to  $f_{B(x,r)}$

page 189, line 12: Change  $x$  to  $x'$

## CHAPTER 5

page 195, line -3: Change to  $\bar{L}(\phi)$  and delete the misplaced overbar

page 197, line -5: Remove “ $f$ ” from the formula for  $\|\partial E\|$

page 201, line -1: Change to “ $\zeta_k(\eta_{\epsilon_k} * \phi)$ ”

page 202, lines 1,2,3: Change all  $|Df|$  to  $\|Df\|$

page 202, line -3: Should be  $\phi \cdot d\mu_k$

page 206, line 3: Should be  $\|Df\|(C_{\epsilon,\delta})$

page 206, line 4: Change subscript to  $\epsilon > 0$

page 207, lines -8 and -10: Change to  $(f_k)_t$

page 208, lines -3: Should be  $B(x, r) \cap U$

page 209, lines 5 and 10:  $H$  should be  $\mathcal{H}$

page 209, lines -9: Add “Let  $\gamma$  give a local parameterization of  $\partial U$  near  $x$ .”

page 209, line 6: Put  $\sigma$  in place of  $\gamma$

page 211, line 1: Change to “... $f \in BV(U)$ . Note that the set  $U$ ...”

page 213, line 13: Less space after “div”

page 216: Add hypothesis that  $n > 1$

page 217, line 11: Should be  $B(x, r) \cap U$

page 219, line -2: Change  $1^*$  to  $1 - \frac{1}{n}$

page 221, line 8: Should be “verify”

page 226, line -1: Change to  $D_r$

page 227, line 2: Should be  $(E \cap B(rL))$

page 227, line -9: Change to  $\mathbb{R}^n$

pages 227-229: This proof contains an error; substitute the correction at the end of this document.

page 230, line 5: Add comma

page 230, line -7: Change “(\*\*\*)” to “Step 4”.

page 234, line -4: Change (ii) to (iii)

page 236, lines 12 and 14: Interchange “Lemma 5.2” and “Lemma 5.5”

page 241, line -7: Interchange  $H_\nu^+$  and  $H_\nu^-$  in formulas for  $\mu(x)$ ,  $\lambda(x)$ .

page 242, lines 2 and 3: Add “ $\lim_{r \rightarrow 0}$ ”

page 242, line 4: Should be “ $0 < r < 1$ ”

page 243, line 3 : Change to  $\{f > M\}$

page 243, lines 3 and 9: Change  $(f - M)$  to  $(f - M)^+$

page 243, line 8: Should be Theorem 5.10

page 244, line 7: Change to (\*\*\*\*)

page 246, line 6: Remove | after the 1

page 246, line 13: Should be  $\int_c^d$

page 249, line 12: Change to  $C_c^1$

page 252, line -9: Should be  $B(z, r)$   
 page 253, line 7: Should be  $(\star\star\star)$   
 page 253, line -10: Add space before  $f^z$

## CHAPTER 6

page 259, line -1: Remove extra )  
 page 260, line 1: Should be  $||[Df]_s|$   
 page 261, line 9: Change  $B(r)$  to  $B(sr)$   
 page 261, line -2: Delete “,(ii)”  
 page 263, line 8: Remove “..”  
 page 263, line 10: Change to “=:  $a$ ”  
 page 264, line 5: Delete ( before  $1 \leq p...$   
 page 269, line -9: Change to  $|y - z|$   
 page 269, line -3: Change to  $B(x, r)$   
 page 269, line -2: Change to (ii)  
 page 270, line -11: Should be (ii)  
 page 270, line -10: Should be (iii)  
 page 272, line 5: Move qed box to end of proof  
 page 272, line 8: Add  $-$  before the first =  
 page 272, line -12: Delete “=  $||D^2 f||_{\perp \Sigma}$ ”.  
 page 272, lines -9 to -11: Delete these lines.  
 page 274, line 2: Should be  $f^\epsilon$   
 page 275, line -3: Change to  $\text{ess sup}$   
 page 276, line -9: Change to  $\text{ess sup}$   
 page 284, line -3. Change to  $B(x, \frac{r}{2^{k+1}})$   
 page 285, line 5: Should be  $R^\lambda$   
 page 287, line -10. Change to  $W^{1,p}(\mathbb{R}^n)$   
 page 288, line 7. Change 6.12 to 6.11

We have been extremely slow in posting these errata that readers have found for the revised edition of our book. Many belated thanks to D Ferizovic, W Ozanski, A Rajapakse and M Safdari for sending lengthy lists of typos, errors and useful comments. Other errors and typos have been found by Daryl Chew, Giovanni Comi, Dengjun Guo, Rene Girard, Seyed Nima Rabiei, Giorgio Stefani and Guanghui Zhang.

Please let us know about any other mistakes you find, at [evans@math.berkeley.edu](mailto:evans@math.berkeley.edu).

**\*\*See the next page for further corrections.\*\***

Correction for page 8, lines 3-7

We need to show also that  $\mathcal{P} \subseteq \mathcal{A}$ . For this, select any  $D \in \mathcal{P}$  and define  $\mathcal{L}' = \{C \subseteq X \mid D \cap C \in \mathcal{S}\}$ . Then  $\mathcal{L}'$  is a  $\lambda$ -system. Furthermore,  $\mathcal{P} \subseteq \mathcal{L}'$ , since  $\mathcal{P}$  is a  $\pi$ -system. Consequently,  $\mathcal{S} \subseteq \mathcal{L}'$ . Thus for all  $D \in \mathcal{P}$ , we have  $D \cap C \in \mathcal{S}$  for all  $C \in \mathcal{S}$ . Consequently,  $D \in \mathcal{A}$ . (Thanks for Guanghui Zhang for finding this error and providing the correction.)

Correction for pages 227-229

4. Claim #1:  $\nu_F = e_n \|\partial F\|$ -a.e.

*Proof of claim:* First note that since  $0 \in \partial^* E$  and  $|\nu_E| = 1 \|\partial E\|$ -a.e., we have

$$\lim_{r \rightarrow 0} \int_{B(r)} |\nu_E - e_n|^2 d\|\partial E\| = 2 \lim_{r \rightarrow 0} \int_{B(r)} 1 - \nu_E \cdot e_n d\|\partial E\| = 0. \quad (\star\star)$$

Let us now write  $\nu_j := \nu_{E_j}$ . Then if  $\phi \in C_c^1(\mathbb{R}^n; \mathbb{R}^n)$ , we have

$$\int_{\mathbb{R}^n} \phi \cdot \nu_j d\|\partial E_j\| = \int_{E_j} \operatorname{div} \phi dy \quad (j = 1, 2, \dots).$$

Since

$$\chi_{E_j} \rightarrow \chi_F \quad \text{in } L_{\text{loc}}^1,$$

we see from the above and  $(\star)$  that

$$\int_{\mathbb{R}^n} \phi \cdot \nu_j d\|\partial E_j\| \rightarrow \int_{\mathbb{R}^n} \phi \cdot \nu_F d\|\partial F\|$$

as  $j \rightarrow \infty$ .

In addition, for all  $\phi$  as above,

$$\int_{\mathbb{R}^n} \phi \cdot \nu_j d\|\partial E_j\| = \frac{1}{s_j^{n-1}} \int_{\mathbb{R}^n} (\phi \circ g_{s_j}) \cdot \nu_E d\|\partial E\|;$$

consequently for  $r > 0$ :

$$\begin{cases} \|\partial E_j\|(B(r)) = \frac{1}{s_j^{n-1}} \|\partial E\|(B(s_j r)) \\ \int_{B(r)} \nu_j d\|\partial E_j\| = \frac{1}{s_j^{n-1}} \int_{B(s_j r)} \nu_E d\|\partial E\|. \end{cases}$$

So  $(\star\star)$  implies

$$\int_{B(r)} |\nu_j - e_n|^2 d\|\partial E_j\| = \int_{B(s_j r)} |\nu_E - e_n|^2 d\|\partial E\| \rightarrow 0,$$

as  $j \rightarrow \infty$ . Select  $\zeta \in C_c^1(\mathbb{R}^n)$  such that  $\zeta \geq 0$ , and put  $\phi = \zeta e_n$  above. Then

$$\int_{\mathbb{R}^n} \zeta e_n \cdot \nu_F d\|\partial F\| = \lim_{j \rightarrow \infty} \int_{\mathbb{R}^n} \zeta e_n \cdot \nu_j d\|\partial E_j\| = \lim_{j \rightarrow \infty} \int_{\mathbb{R}^n} \zeta d\|\partial E_j\|. \quad (\star\star\star)$$

Choose now any radius  $r > 0$  for which  $\|\partial F\|(\partial B(r)) = 0$ . Pick  $h > 0$  and select the function  $\zeta$  above so that  $0 \leq \zeta \leq 1$ ,  $\zeta \equiv 1$  on  $B(r)$ ,  $\zeta \equiv 0$  on  $\mathbb{R}^n - B(r+h)$ . Then lower semicontinuity and  $(\star\star\star)$  imply

$$\|\partial F\|(B(r)) \leq \int_{B(r+h)} \zeta e_n \cdot \nu_F d\|\partial F\|.$$

Sending  $h \rightarrow 0$ , we find that

$$\|\partial F\|(B(r)) \leq \int_{B(r)} e_n \cdot \nu_F d\|\partial F\|.$$

for all  $r$  as above. Since  $e_n \cdot \nu_F \leq 1$ , it follows that  $e_n \cdot \nu_F = 1$   $\|\partial F\|$ -a.e. and so the claim holds.

We also see from the above that

$$\|\partial F\|(B(r)) = \lim_{j \rightarrow \infty} \|\partial E_j\|(B(r))$$

whenever  $\|\partial F\|(\partial B(r)) = 0$ .