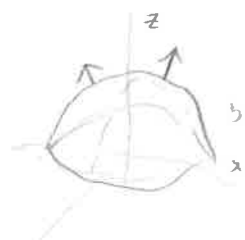
Math 53 DIS 108/109
Quiz: May 1, 2015 Name: Key

Show your work fully for all questions. Quiz has **front** and **back** sides.

Problem 1: Evaluate the integral $\iint_S (\nabla \times F) \cdot \mathbf{dS}$ where $F(x, y, z) = (2y\cos(z), e^x \sin(z), xe^y)$ and S is the hemisphere $x^2 + y^2 + z^2 = 9$, $z \geq 0$, oriented upward.

By Stokes theorem $\iint_S (\nabla \times F) \cdot d\vec{S} = \int_C F \cdot dr$

$= \int_C F \cdot r' \, dt$



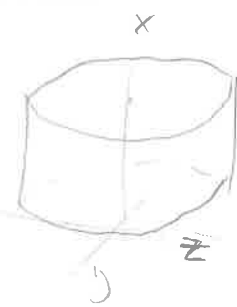
let $r = (3\cos\theta, 3\sin\theta, 0)$

$r' = (-3\sin\theta, 3\cos\theta, 0)$

$F(r(\theta)) = (-2 \cdot 3\sin\theta \cdot \cos(0), e^{3\cos\theta} \sin 0, 3\cos\theta \, e^{3\sin\theta})$

$\int_0^{2\pi} F \cdot r' dt = \int_0^{2\pi} -18 \cdot \sin^2\theta = \int_0^{2\pi} -18\left(\frac{1 - \cos 2\theta}{2}\right) = -18\pi$

Problem 2: Evaluate the integral $\iint_S F \cdot \mathbf{dS}$ where $F(x, y, z) = (x^2, -y, z)$ and S is the surface of the region $y^2 + z^2 \leq 9$, $0 \leq x \leq 2$



By the divergence thm $\iint_S F \cdot dS = \iiint_E \operatorname{div} F \, dV$

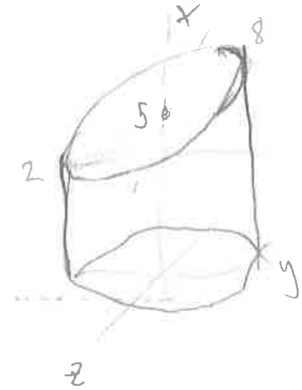
So $\int_0^2 \int_{-3}^{3} \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} 2x - 1 + 1 \, dz \, dy \, dx$

Convert y, z to cylindrical

$\int_0^2 \int_0^{2\pi} \int_0^3 2x \, r \, dr \, d\theta = \int_0^2 2x \cdot 2\pi \cdot \frac{3^2}{2} = 9\pi \cdot \frac{2^2 \cdot 2}{2} = 36\pi$

This was intentionally a <u>hard</u> problem

Problem 3: Evaluate the surface integral $\iint_S xz\,dS$ where S is the boundary of the region enclosed by the cylinder $y^2+z^2=9$, and the planes $x=0$ and $x+y=5$



Standard Way:

We have three surfaces, top, bottom, side

We must parameterize each and integrate

$$\iint_D f(u,v)\,|r_u \times r_v|\,du\,dv$$

Bottom: $z = u\cos v$, $y = u\sin v$, $x = 0$

$$\int_D u\cos v \cdot 0 \cdot |r_u \times r_v| = 0$$

Top: $z = u\cos v$, $y = u\sin v$, $x = 5 - u\sin v$

$$|r_u \times r_v| = \begin{vmatrix} i & j & k \\ -\sin v & \sin v & \cos v \\ -u\cos v & u\cos v & -u\sin v \end{vmatrix}$$

$$= \left| -u(\sin^2 v + \cos^2 v)i + -u(\cos^2 v + \sin^2 v)j + u(\sin v\cos v - \sin v\cos v)k \right|$$

$$= \sqrt{u^2+u^2} = \sqrt{2}\,u$$

So $\displaystyle\int_0^{2\pi}\int_0^3 (5 - u\sin v)\,u\cos v\,\sqrt{2}\,u\,du\,dv$

$$\int_0^{2\pi} 5\cdot\frac{3^3}{3}\sqrt{2}\cos v - \frac{3^4}{4}\sqrt{2}\sin v\cos v\,dv$$

$t = \sin v$

$$= 0 + \int_{v=0}^{v=2\pi} \frac{3^4}{4}\sqrt{2}\,t\,dt$$

$$= 0$$

Side: $z = 3\cos\theta$, $y = 3\sin\theta$, $x = x$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 3\sin\theta & 3\cos\theta \end{vmatrix} = 3$$

$$\int_0^{2\pi}\int_0^{5-3\sin\theta} x \cdot 3\cos\theta \cdot 3\,dx\,d\theta$$

$$\int_0^{2\pi} 9\cos\theta\,\frac{(5-3\sin\theta)^2}{2}\,d\theta \quad \text{let } t = 5 - 3\sin\theta,\ dt = -3\cos\theta$$

$$= 0$$

So the sum integral $= 0$. We can also find this <u>much</u> faster using z symmetry.