

Show your work fully for all questions. Quiz has **front** and **back** sides.

Problem 1: Let $f(x, y, z) = 3x^2 + yz$ and $\mathbf{F}(x, y, z) = x\mathbf{i} + 2\sin(y)\mathbf{j} + xz\mathbf{k}$. For each of the following expressions either evaluate the expression or state why it is not meaningful.

• $\nabla \times (\nabla f)$

$$\nabla \times (\nabla f) = 0$$

• $\text{div}(f)$ Meaningless: div maps vectors to scalars

• $\text{curl}(\mathbf{F})$
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & 2\sin y & xz \end{vmatrix} = \mathbf{i}(0-0) + \mathbf{j}(0-z) + \mathbf{k}(0-0) = -z\mathbf{j}$$

• $\nabla \cdot (\nabla f)$

$$\nabla \cdot \nabla f = f_{xx} + f_{yy} + f_{zz} = 6 + 0 + 0 = 6$$

Problem 2: Find a parametric representation for the part of the hyperboloid $9x^2 - 9y^2 - z^2 = 9$ that lies in front of the yz -plane.

$$9x^2 = 9 + 9y^2 + z^2$$

$$x = \sqrt{1 + y^2 + \frac{1}{9}z^2}$$

(negative square root only since we're in front of yz)

Now let $y = u$

$$z = v$$

$$x = \sqrt{1 + u^2 + \frac{1}{9}v^2}$$

(In front is slightly ambiguous so I'll accept either + or - but not \pm)

$$-\infty < u < \infty$$

$$-\infty < v < \infty$$

Problem 3: Find the area of the part of the paraboloid $x = y^2 + z^2$ that lies in the cylinder $y^2 + z^2 = 16$.

We can use the formula

$$SA = \iint_D \sqrt{1 + \frac{\partial x}{\partial y}^2 + \frac{\partial x}{\partial z}^2} dA$$

$$= \iint_D \sqrt{1 + 4y^2 + 4z^2} dA$$

This is a good place for polar coordinates

$$= \int_0^{2\pi} \int_0^4 \sqrt{1 + 4r^2} r dr d\theta$$

$$= 2\pi \int_0^4 r \sqrt{1 + 4r^2} dr \quad \text{let } u = 1 + 4r^2 \\ du = 8r dr$$

$$= 2\pi \int_1^{1+4 \cdot 4^2} \sqrt{u} \frac{du}{8}$$

$$= \frac{\pi}{4} \left(\frac{2}{3} u^{3/2} \Big|_1^{65} \right) = \frac{\pi}{6} (65^{3/2} - 1)$$

There are many
ways to solve this
one