Math 53 DIS 108/109
Quiz: April 8, 2015

Name: Key

Show your work fully for all questions. Quiz has **front** and **back** sides.

Problem 1: Find the gradient vector field for the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$

$$f_x = 2x \cdot \frac{1}{2\sqrt{x^2+y^2+z^2}}$$

$$f_y = 2y \cdot \frac{1}{2\sqrt{x^2+y^2+z^2}}$$

$$f_z = 2z \cdot \frac{1}{2\sqrt{x^2+y^2+z^2}}$$

$$\nabla f = (x, y, z) * \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Problem 2: Evaluate the integral $\int_C \frac{x}{y} ds$, where C is the curve $x = y^2$ from $(0, 0)$ to $(1, 1)$.

let $y = t$, t from 0 to 1, $y = t$, $x = t^2$ $\quad \frac{dy}{dt} = 1 \quad \frac{dx}{dt} = 2t$

$$\int_0^1 \frac{t^2}{t} \sqrt{1^2 + (2t)^2}\, dt = \int_0^1 t\sqrt{1+4t^2}\, dt$$

let $u = 1+4t^2$, $du = 8t\,dt$

So $$\int_{u=1}^{u=5} \frac{\sqrt{u}}{8} du = \frac{2}{3}\frac{u^{3/2}}{8} = \frac{1}{12}\left(5^{3/2} - 1\right)$$

Problem 3: Evaluate the integral $\int_C F \cdot dr$, for the vector field $F(x,y) = x^2 i + y^2 j$ and C is the arc of the parabola $y = 2x^2$ from $(-1,2)$ to $(2,8)$.

We observe $\frac{d}{dy} x^2 = \frac{d}{dx} y^2$ on $\mathbb{R}^n$, so F is ∇f for some f.

$f = \frac{x^3}{3} + \frac{y^3}{3}$ so $\int_C F \cdot dr = f(2,8) - f(-1,2)$

$$= \frac{2^3}{3} + \frac{8^3}{3} - \frac{(-1)^3}{3} - \frac{2^3}{3}$$

$$= \frac{8^3}{3} + \frac{1}{3}$$ (simplification unnecessary)

$$= \frac{513}{3}$$

$$= 171$$