Math 53 DIS 108/109
Quiz: April 3, 2015

Name: Key

Show your work fully for all questions. Quiz has **front** and **back** sides.

Problem 1: evaluate $\iiint_E e^{(x^2+y^2+z^2)^{\frac{3}{2}}}\,dV$ where E is the portion of the unit ball $x^2+y^2+z^2 \le 1$ that lies in the first octant.

We convert to spherical coords.

First octant $\to$ $0 \le \theta \le \pi/2$
$0 \le \phi \le \pi/2$

Ball $\to$ $0 \le \rho \le 1$

thus we have $\int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 e^{(\rho^2)^{3/2}} \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$

let $u = \rho^3$, $du = 3\rho^2\, d\rho$ $\quad \rho = 0 \to u = 0$, $\rho = 1 \to u = 1$

$$\to \int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 e^u \sin\phi \frac{du}{3}\, d\theta\, d\phi$$

$$= \int_0^{\pi/2}\int_0^{\pi/2} \frac{(e^1 - e^0)}{3} \sin\phi \, d\theta\, d\phi$$

$$= \int_0^{\pi/2} \frac{\pi}{2}\frac{e-1}{3} \sin\phi\, d\phi$$

$$= \left(\frac{\pi}{2}\frac{e-1}{3}\right)\left(-\cos\phi \Big|_0^{\pi/2}\right) = \frac{\pi}{2}\frac{e-1}{3}$$

Problem 2: Use the transformation $u = xy$, $v = xy^2$ to evaluate $\iint_R y^2 dA$ where R is the region bounded by the curves $xy = 1$, $xy = 2$, $xy^2 = 1$, $xy^2 = 2$.

First solve for x and y

$$x = u^2/v$$

$$y = v/u$$

$$\text{Jacobian} = \det \begin{pmatrix} 2u/v & -u^2/v^2 \\ -v/u^2 & 1/u \end{pmatrix} = 2\frac{u}{v}\frac{1}{u} - \left(-\frac{v}{u^2}\right)\left(-\frac{u^2}{v^2}\right)$$

$$= \frac{1}{v}$$

Our bounds conveniently become $u=1$, $u=2$, $v=1$, $v=2$

So our integral is

$$\int_1^2 \int_1^2 v^2/u^2 \cdot \frac{1}{v} \, du \, dv$$

$$\int_1^2 -\frac{v}{u} \Big|_1^2 \, dv$$

$$\int_1^2 v\left(1 - \frac{1}{2}\right) dv$$

$$= \frac{1}{4} v^2 \Big|_1^2 = 3/4$$