Math 53 DIS 108/109
Quiz: March 18, 2015

Name: Key

Show your work fully for all questions. Quiz has **front** and **back** sides.

Problem 1: Find the mass and center of mass of the lamina bounded by $y = x^2$ and $x = y^2$ with density $\rho(x, y) = \sqrt{y}$.

$$M = \int_0^1 \int_{x^2}^{\sqrt{x}} \sqrt{y}\, dy\, dx$$

$$= \int_0^1 \frac{2}{3} y^{3/2} \Big|_{x^2}^{x^{1/2}} = \int_0^1 \frac{2}{3}\left(x^{3/4} - x^3\right)$$

$$= \left(\frac{2}{3}\cdot\frac{4}{7} x^{7/4} - \frac{2}{3}\cdot\frac{1}{4} x^4\right)\Big|_0^1 = \boxed{\frac{2}{3}\left(\frac{4}{7} - \frac{1}{4}\right) = M} = \frac{3}{14}$$

$$M_x = \int_0^1 \int_{x^2}^{\sqrt{x}} \sqrt{y}\cdot y\, dy\, dx = \int_0^1 \frac{2}{5} y^{5/2}\Big|_{x^2}^{x^{1/2}} = \int_0^1 \frac{2}{5}\left(x^{5/4} - x^5\right) = \frac{2}{5}\left(\frac{4}{9}x^{9/4} - \frac{1}{6}x^6\right)\Big|_0^1$$

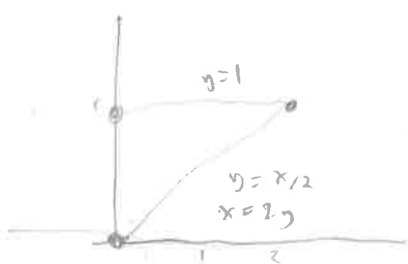
$$= \frac{2}{5}\left(\frac{4}{9} - \frac{1}{6}\right)$$

$$M_y = \int_0^1 \int_{x^2}^{\sqrt{x}} \sqrt{y}\cdot x\, dy\, dx = \int_0^1 x\, \frac{2}{3} y^{3/2}\Big|_{x^2}^{x^{1/2}} = \int_0^1 \frac{2}{3}\left(x^{7/4} - x^4\right)$$

$$= \frac{2}{3}\left(\frac{4}{11}x^{11/4} - \frac{1}{5}x^5\right)\Big|_0^1$$

$$\boxed{\left(\frac{28}{55}, \frac{14}{27}\right)} = \text{Center of Mass} = \left(\frac{2}{3}\left(\frac{4}{11} - \frac{1}{5}\right), \frac{2}{5}\left(\frac{4}{9} - \frac{1}{6}\right)\right) \Big/ \left(\frac{2}{3}\left(\frac{4}{7} - \frac{1}{4}\right)\right) = \frac{2}{3}\left(\frac{4}{11} - \frac{1}{5}\right)$$

Problem 2: Find the area of the part of the surface $z = 1 + 3x + 2y^2$ that lies above the triangle with vertices $(0, 0)$, $(0, 1)$, and $(2, 1)$.



$$SA = \iint \sqrt{1 + f_x^2 + f_y^2}\, dA$$

$f_x = 3$

$f_y = 4y$

$$SA = \int_0^1 \int_0^{2y} \sqrt{1 + 3^2 + 16y^2}\, dx\, dy$$

$$= \int_0^1 \sqrt{10 + 16y^2}\, x\Big|_0^{2y} dy$$

$$= 2\int_0^1 \sqrt{10 + 16y^2}\, y\, dy$$

let $u = 10 + 16y^2$

$du = 32y\, dy$

$$= \frac{1}{16}\int_0^1 \sqrt{u}\, du = \frac{1}{16}\left(\frac{2}{3}u^{3/2}\right)\Big|_0^1$$

$$= \frac{1}{16}\cdot\frac{2}{3}\cdot\left((10 + 16)^{3/2} - 10^{3/2}\right)$$

Problem 3: Evaluate the integral $\iiint_R e^{z/y}$, for $R = \{(x,y,z) | y \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq xy\}$

$$\int_0^1 \int_y^1 \int_0^{xy} e^{z/y} \, dz\, dx\, dy$$

$$= \int_0^1 \int_y^1 y e^{z/y} \Big|_0^{xy} \, dx\, dy$$

$$= \int_0^1 \int_y^1 y e^x - y \, dx\, dy$$

$$= \int_0^1 (y e^x - yx) \Big|_y^1 \, dy$$

$$= \int_0^1 (ye - y - ye^y + y^2)$$

Now $\int ye^y = ye^y - e^y$ by $\int$ by parts

so $\left(\frac{y^2}{2}(e-1) + \frac{y^3}{3} - ye^y + e^y \right) = \frac{1}{2}(e-1) + \frac{1}{3} - e + e - (0 + 0 - 0 + 1)$

$$= \frac{1}{2}e - \frac{7}{6}$$