Math 53 DIS 108/109
Quiz: March 11, 2015

Name: Key

Show your work fully for all questions. Quiz has **front** and **back** sides.

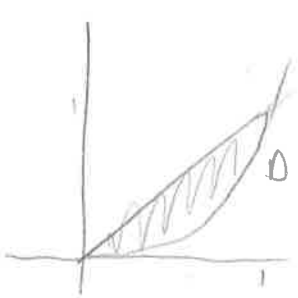
Problem 1: Find the volume of the solid bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$

The volume between the two surfaces is the integral of the difference, that is $\iint (4-x^2-y^2) - 3x^2 - 3y^2 = 4\iint (1 - x^2 - y^2)$, this hits the xy plane at $x^2+y^2 = 1$ i.e. a circle of radius 1. Thus we have

$$4\int_0^{2\pi}\int_0^1 (1-r^2)\, r\, dr\, d\theta$$

$$= 4\int_0^{2\pi} \left(\frac{r^2}{2} - \frac{r^4}{4}\right)\Big|_0^1 d\theta = 4\left(\frac{1}{2} - \frac{1}{4}\right)\cdot 2\pi = 2\pi$$

Problem 2: Evaluate the integral $\iint_D (x^2 + 2y)dA$, for D bounded by $y = x$, $y = x^3$, $x \geq 0$.



$$\int_0^1\int_{x^3}^{x} (x^2+2y)\, dy\, dx$$

$$= \int_0^1 (x^2 y + y^2)\Big|_{x^3}^{x} dx = \int_0^1 -x^5 - x^6 + x^3 + x^2\, dx$$

$$= \left(\frac{-x^7}{7} + \frac{-x^6}{6} - \frac{x^4}{4} - \frac{x^3}{3}\right)\Big|_0^1$$

$$= -\frac{1}{7} + \frac{-1}{6} - \frac{1}{4} - \frac{1}{3}$$

$$= -\frac{1}{7} + \frac{1}{4} + \frac{1}{6}$$

$$= \frac{-1}{7} + \frac{5}{12}$$

$$= \frac{23}{82}$$

Problem 3: Evaluate the integral $\iint_R \frac{2x}{1+xy} dA$, for $R = \{(x,y) | 0 \le x \le 1, 0 \le y \le 1\}$

This is much easier if we integrate by y first

$$= 2 \int_0^1 \int_0^1 \frac{x}{1+xy} \, dy \, dx$$

let $u = 1+xy$, $du = x$

$$= 2 \int_0^1 \int \frac{du}{u} \, dx$$

$$= 2 \int_0^1 \ln(1+xy) \Big|_0^1 \, dx$$

$$= 2 \int_0^1 \ln(1+x) \, dx$$

Recall

$$\frac{d}{dx} x\ln(x) - x = \ln x + \frac{x}{x} - 1 = \ln x$$

$$\text{so } = 2 \left(\ln(1+x)(1+x) - (1+x) \right) \Big|_0^1$$

$$= 2 \cdot \left(\ln(2) \cdot 2 - 2 - \ln(1)(1) + 1 \right)$$

$$= 2(\ln 2 - 1)$$