Math 53 DIS 108/109
Quiz: March 4, 2015

Name: Key

Show your work fully for all questions. Quiz has **front** and **back** sides.

Problem 1: Find the absolute maximum and minimum of $f(x, y) = 2x + 3y - x^2 - y^2$ in the region $x^2 + 2y^2 \leq 9$.

We need to check interior and boundary

Interior: $f_x = 2 - 2x \to f_x = 0$ when $x = 1$

$f_y = 3 - 2y \to f_y = 0$ when $y = 1.5$

($1^2 + 1.5^2 < 9$ so this is inside our region)

We employ the D-test $f_{xx} = -2$, $f_{yy} = -2$, $f_{xy} = 0$

$D = 4 > 0$, $f_{xx} < 0$ so $f(1, 1.5) = 2 + 3 \cdot 1.5 - 1 - 2.25$

$= 3.25$ is a local max

Now we check boundaries:

We can use Lagrange multipliers

$\nabla f = \lambda \nabla g \to 2 - 2x = \lambda \cdot 2x$

$3 - 2x = \lambda \cdot 4y$

$x^2 + 2y^2 = 9$

or substitute $x = \pm\sqrt{9 - 4y^2}$

either of these is worth full credit

Unfortunately I made a mistake in tuning the parameters so the resulting equation is really hard to solve (It uses the quartic formula)

In the end the boundary max is $f(1.34\ldots, 1.96\ldots) = 3.017\ldots$

and min is $f(-2.74\ldots, -0.87\ldots) = -16.325\ldots$

Problem 2: Find the extreme values of $f(x, y, z) = y - 2x$ subject to the constraints $x + y + z = 3$ and $x^2 + z^2 = 4$

We'll solve this with Lagrange multipliers.

let $g(x,y,z) = x+y+z$, $h(x,y,z) = x^2+z^2$

Then our Lagrange condition is $\nabla f(x,y,z) = \lambda \nabla g(x,y,z) + \mu \nabla h(x,y,z)$

So we have this system:

a) $-2 = \lambda \cdot 1 + \mu \cdot 2x$

b) $1 = \lambda \cdot 1 + \mu \cdot 0$

c) $0 = \lambda \cdot 1 + \mu \cdot 2$

d) $x+y+z = 3$

e) $x^2 + z^2 = 4$

$b \Rightarrow \lambda = 1$

$a \Rightarrow x = \frac{-3}{2\mu}$

$c \Rightarrow z = \frac{-1}{2\mu}$

$\rightarrow x = 3z$

$e) \rightarrow 9z^2 + z^2 = 4$

$z = \pm \frac{2}{\sqrt{10}}$

$x = \pm \frac{6}{\sqrt{10}}$

d) $y = 3 \mp \frac{8}{\sqrt{10}}$

So our points are

$P_1 \left(\frac{6}{\sqrt{10}}, \frac{2}{\sqrt{10}}, 3 - \frac{8}{\sqrt{10}}\right)$

and

$P_2 \left(\frac{-6}{\sqrt{10}}, \frac{-2}{\sqrt{10}}, 3 + \frac{8}{\sqrt{10}}\right)$

At P_1 $f = y - 2x = 3 - \frac{20}{\sqrt{10}}$

and at P_2

$f = 3 + \frac{20}{\sqrt{10}}$

So these are our extreme values.