Math 53 DIS 108/109
Quiz: February 20, 2015

Name: Key

Show your work fully for all questions. Quiz has **front** and **back** sides.

Problem 1: Find the first partial derivatives of $f(x, y) = \frac{x}{(x+y)^2}$. Is it differentiable at the point $(0, 1)$? Why or why not?

$$f_x = \frac{(x+y)^2 \cdot 1 - x \cdot 2(x+y)}{(x+y)^{2\,2}} = \frac{(x+y)(y-x)}{(x+y)^4}$$

$$f_y = \frac{(x+y)^2 \cdot 0 - x \cdot 2 \cdot (x+y)}{(x+y)^4} = \frac{(x+y)(-2x)}{(x+y)^4}$$

Since $x+y \neq 0$ at $(0,1)$ f_x and f_y are continuous at that point, thus f is differentiable at $(0,1)$

Problem 2: Give the linearization $L(x, y)$ of the function $f(x, y) = \sqrt{x + e^{4y}}$ at the point $(3, 0)$.

$$L(x,y) = f_x(3,0) \cdot (x-3) + f_y(3,0)(y-0) + f(3,0)$$

$$f_x = \frac{1}{2\sqrt{x+e^{4y}}} \cdot 1 \quad , \quad f_y = \frac{1}{2\sqrt{x+e^{4y}}} 4e^{4y}$$

$$f_x(3,0) = \frac{1}{2\sqrt{3+1}} = \frac{1}{4} \qquad f_y(3,0) = \frac{1}{2\sqrt{3+1}} \cdot 4 \cdot 1 = 1$$

$$f(3,0) = \sqrt{3+1} = 2$$

$$L(x,y) = \frac{1}{4}(x-3) + 1 \cdot (y-0) + 2$$

Problem 3: Find the equation for the tangent plane to the surface $z = y\sin(x+2y)$ at the point $(-2, 1, 0)$.

The tangent plane is given by

(for $z = f(x,y)$)

$$(z - z_0) = f_x (x - x_0) + f_y (y - y_0)$$

$f_x = y\cos(x+2y)$, $f_y = 2y\cos(x+2y) + \sin(x+2y)$

$f_x(-2,1) = 1 \cdot \cos(-2+2\cdot 1) = 1$

$f_y(-2,1) = 2 \cdot 1 \cdot \cos(-2+2\cdot 1) + \sin(-2+2\cdot 1) = 2$

So

$$z - 0 = 1\cdot(x+2) + 2\cdot(y-1) = x + 2y$$