Math 53 DIS 108/109
Quiz: February 11, 2015

Name: Key

Show your work fully for all questions. Quiz has **front** and **back** sides.

Problem 1: Find the curve of intersection of these two surfaces; give a vector function describing that curve: The hyperboloid $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$.

Let $x = \cos t$, $y = \sin t$

this traces the circle

$x^2 + y^2 = 1$

Then let $z = \cos^2 t - \sin^2 t$

so

$r(t) = (\cos t, \sin t, \cos^2 t - \sin^2 t)$

Or

$x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$

let $x = t$ $\quad (-1 \le t \le 1)$

$y^2 = 1 - t^2$

$z = t^2 - (1 - t^2)$

so our curve is given by

$r(t) = (t, \pm\sqrt{1-t^2}, 2t^2 - 1) \quad (-1 \le t \le 1)$

Note this is technically two curves

Problem 2: Find the derivative of the vector functionc $\mathbf{r}(t) = (\tan t, \sec t, 1/t^2)$

$$r'(t) = \left(\frac{d}{dt}\tan(t), \frac{d}{dt}\sec(t), \frac{d}{dt}\frac{1}{t^2}\right)$$

$\frac{\sin(t)}{\cos^2(t)}$

$$= \left(\sec^2(t), \tan(t)\sec(t), -2\cdot\frac{1}{t^3}\right)$$

Problem 3: Find the limit if it exists or show it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 + y^2} \quad \star$$

Top is higher power than bottom, so we suspect 0.

Now $0 \le \frac{x^4}{x^2+y^2} \le \frac{x^4}{x^2} = x^2$ which goes to 0, so by the squeeze thm

$\frac{x^4}{x^2+y^2} \to 0$

Similarly $0 \ge \frac{-y^4}{x^2+y^2} \ge \frac{-y^4}{y^2} = -y^2$ which goes to 0, so by the

squeeze theorem $\frac{-y^4}{x^2+y^2} \to 0$

Thus, as the sum of two limits, $\lim_{x,y\to 0,0} \frac{x^4-y^4}{x^2+y^2} = 0+0=0$

Problem 4: Find the limit if it exists or show it does not exist.

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz}{x^2 + y^2 + z^2}$$

Suppose we go to 0 along $y=0$, $x=z$ then we have $\lim_{x\to 0} \frac{0}{x^2+x^2} = 0$

But if we go along $x=y=z$ then we have $\lim_{x\to 0} \frac{x^2+x^2}{x^2+x^2+x^2} = \frac{2}{3}$

Since we get different answers for different curves, the limit

does not exist

$\star$ I've written the straightforward way of doing 3. The clever way is

recognizing $\frac{x^4-y^4}{x^2+y^2} = \frac{(x^2+y^2)(x^2-y^2)}{(x^2+y^2)}$ which is the product of

two functions with defined limits (1 and 0) so the limit is 0